Communication against restricted adversaries: between Shannon and Hamming

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Anand D. Sarwate

Rutgers University

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Reliable communication, revisited

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Let's zoom in on binary channels with erasures.

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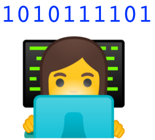

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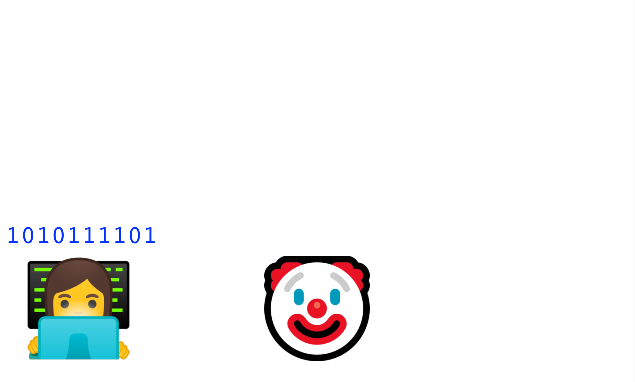

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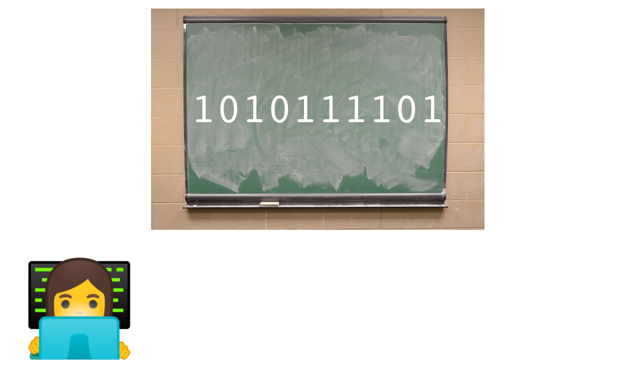

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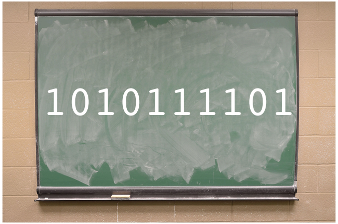

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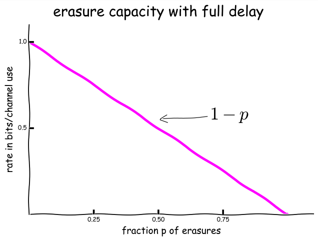
The erasure channel: adversarial vs. random



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And we can achieve it many different ways.

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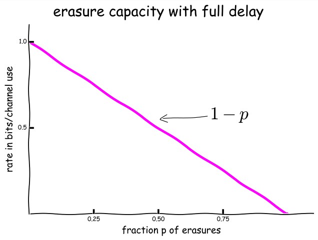

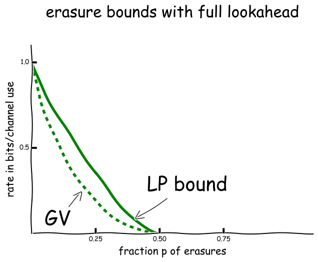

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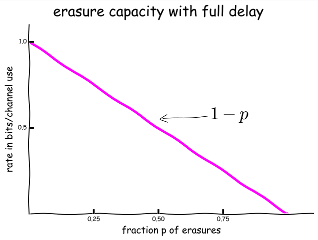

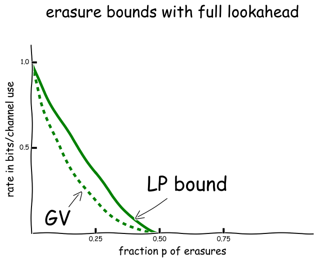

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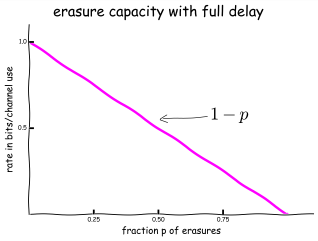

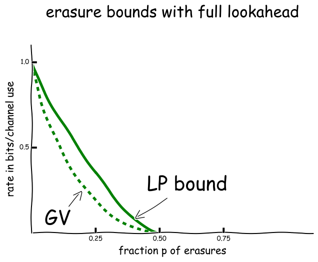

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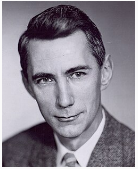

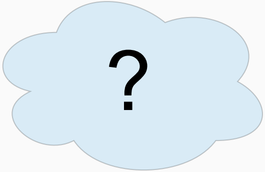

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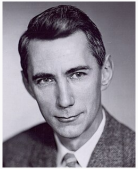

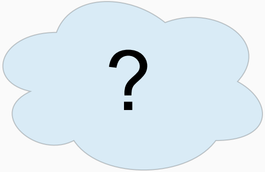

We are suggesting a different line of attack:

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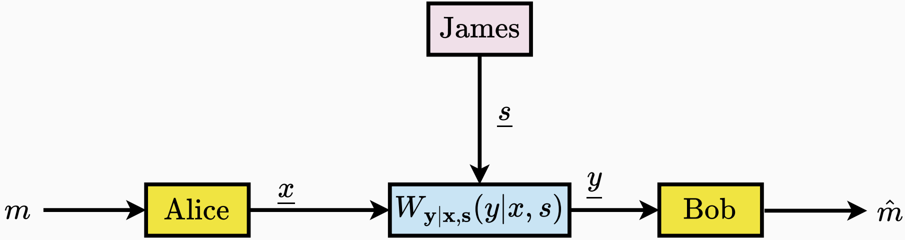
We are suggesting a different line of attack:

1. Use **arbitrarily varying channels (AVCs)** to develop a **unified framework** for both the Shannon and Hamming models.
2. Explore **intermediate models** to see **what lies in the the gap**.
3. Discover **coding strategies** and **new attacks/converses** to see what **resources are needed** to communicate reliably.

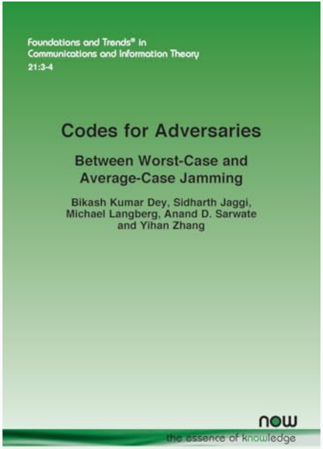
AVCs model channel “noise” as a state variable



In an **adversarial channel model**, **Alice** wants to communicate with **Bob** over a channel whose time-varying state is controlled by an adversarial **jammer** James.

- Alice and James may be **constrained** in how they communicate.
- Capacity depends on **what James knows** about m and $\underline{x}$.

Shameless self-promotion



We have a monograph (December 2024!) on coding against adversarial interference in a variety of settings using the framework of **arbitrarily varying channels**:

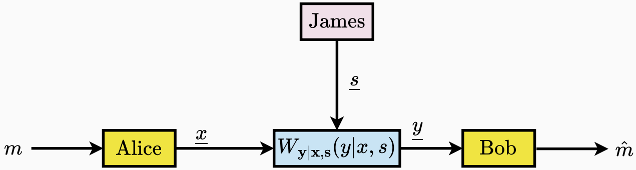
- ✔ Unified treatment of random noise (Shannon-theoretic) and worst-case noise (coding-theoretic).
- ✔ Intermediate models for jammers who can eavesdrop: online and myopic.
- ✔ Examples, open problems, and more!

What's coming up next

1. Arbitrarily varying channels (AVCs)
2. Some key ingredients
3. Causal adversarial models
4. Myopic adversarial models
5. Computationally efficient codes for causal adversaries
6. Looking forward

Arbitrarily varying channels (AVCs)

The basic channel model



Let $\mathcal{X}$, $\mathcal{S}$, and $\mathcal{Y}$ be discrete alphabets. An AVC is a discrete channel $W_{\mathbf{y}|\mathbf{x},\mathbf{s}}(y|x,s)$ such that

$$W_{\underline{\mathbf{y}}|\underline{\mathbf{x}},\underline{\mathbf{s}}}(\underline{y}|\underline{x},\underline{s}) = \prod_{i=1}^{n} W_{\mathbf{y}|\mathbf{x},\mathbf{s}}(y_i|x_i,s_i)$$

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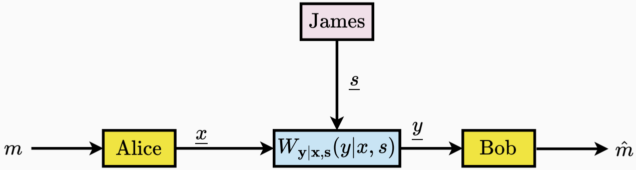

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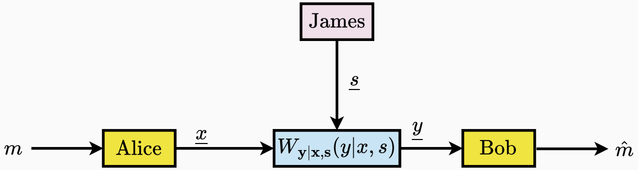

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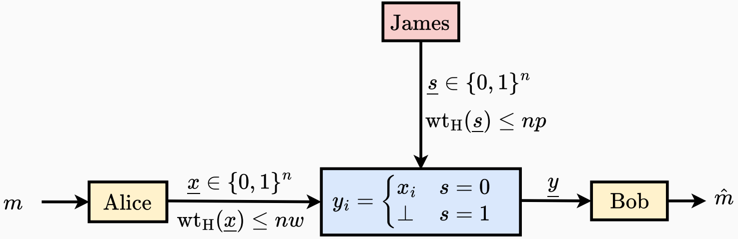
Examples: For binary channels $\underline{s}$ could be the error erasure pattern.

Input and cost constraints for AVCs

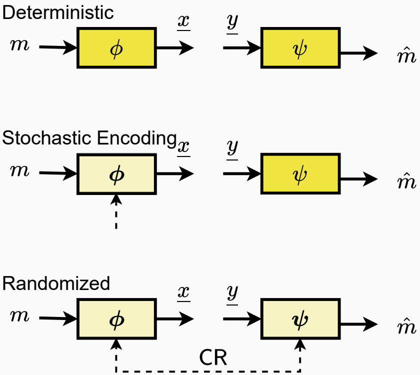
We impose that the types $T_{\underline{x}}$ and $T_{\underline{s}}$ of the codeword $\underline{x}$ and the state $\underline{s}$ lie be in convex subsets of the probability simplices $\Delta(\mathcal{X})$ and $\Delta(\mathcal{S})$:

$$T_{\underline{x}} \in \Gamma \subseteq \Delta(\mathcal{X})$$
$$T_{\underline{s}} \in \Lambda \subseteq \Delta(\mathcal{S})$$

Example: For binary channels $\underline{x}$ and $\underline{s}$ have bounded Hamming weight.



Defining codes and input constraints



An (n, M, Γ) code is

$$\phi : [M] \to \mathcal{X}^n \qquad \text{(encoder)}$$
$$\psi : \mathcal{Y}^n \to [M] \qquad \text{(decoder)}$$

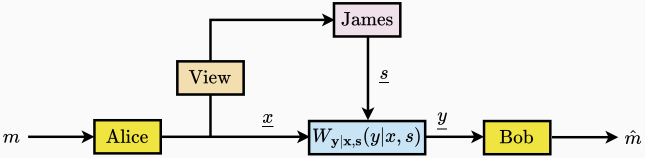
such that

$$T_{\phi(m)} \in \Gamma$$

The rate is $R = \frac{1}{n} \log_2(M)$.

A **randomized code** lets Alice and Bob choose their code in secret. If Alice and Bob do not share common randomness, Alice can still use **stochastic encoding**.

What James knows: Shannon, Hamming, and in between



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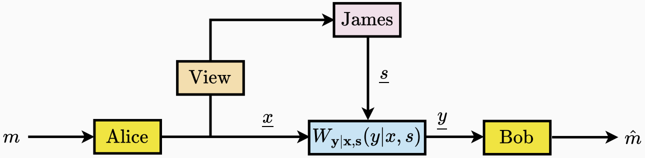

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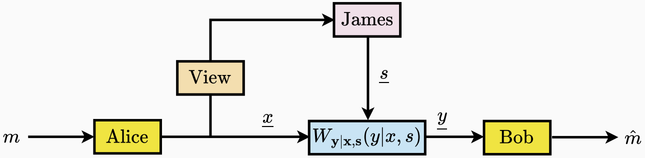

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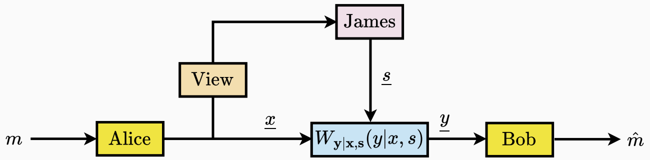

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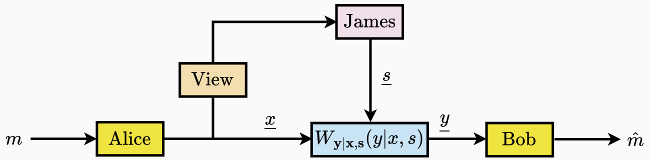

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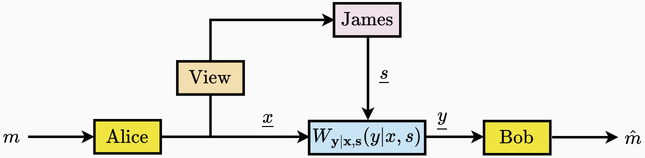

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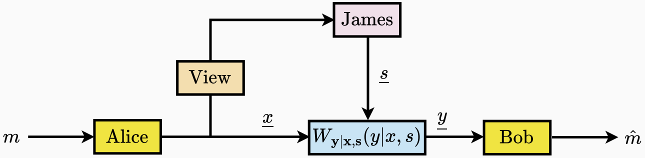

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Maximal error and capacity

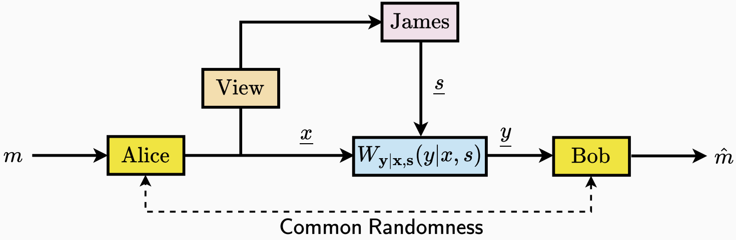
Maximal and average error:

$$P_{\mathrm{err}}(\phi, \psi) = \max_{\text{jamming strategies}} \frac{1}{M} \sum_{m=1}^{M} \sum_{\mathbf{x} \in \mathcal{X}^n} \mathbb{P}\left(\psi(\mathbf{y}) \neq m \mid \mathbf{x}\right) \mathbb{P}_{\phi}\left(\phi(m) = \mathbf{x}\right)$$

A rate R is achievable if for any $\epsilon > 0$ there exists an infinite sequence of rate R codes whose maximal probability of error is $< \epsilon$.
Let C_{obl} and C_{omni} be the capacities for oblivious and omniscient adversaries. In general

$$(\textit{Hamming}) \qquad C_{\mathrm{omni}} \qquad \leq \qquad C_{\mathrm{obl}} \qquad (\textit{Shannon})$$

Common randomness makes the problem easier



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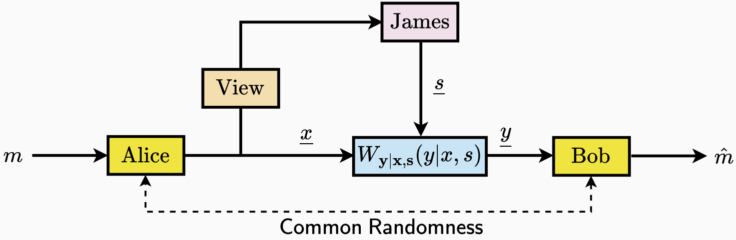

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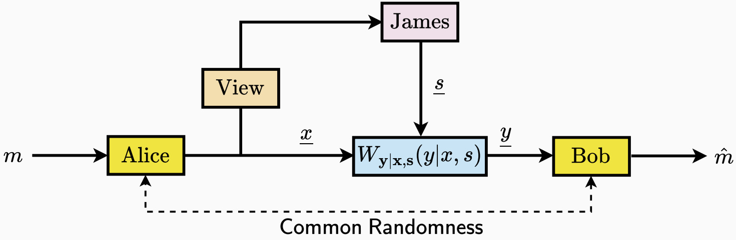

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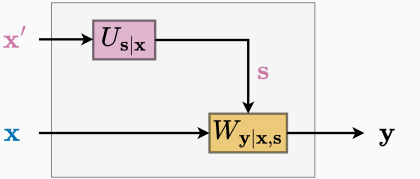
- **Omniscient:** find $\sum_s W_{\mathbf{y}|\mathbf{x},\mathbf{s}}(y|x,s)U_{\mathbf{s}|\mathbf{x}}(s|x)$ with lowest Shannon capacity.

Deterministic codes and ECN Symmetrizability



An AVC is **Ericson-Csiszár-Narayan (ECN) symmetrizable** if James can spoof Alice's codeword. That is, for all $(\mathbf{y}, \mathbf{x}, \mathbf{x}')$, we have

$$\sum_{s} U_{\mathbf{s}|\mathbf{x}'} W_{\mathbf{y}|\mathbf{x},\mathbf{s}} = \sum_{s} U_{\mathbf{s}|\mathbf{x}} W_{\mathbf{y}|\mathbf{x}',\mathbf{s}}.$$

Without common randomness, the capacity of a symmetrizable AVC $C_{\mathrm{obl}} = 0$.

Intermediate model 1: James gets delayed information

Delayed: James cannot use the full codeword in his strategy

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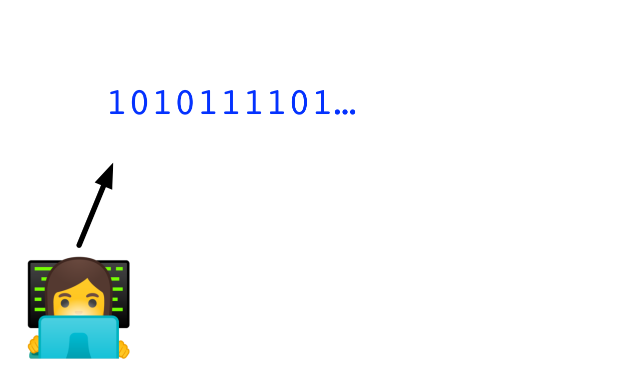

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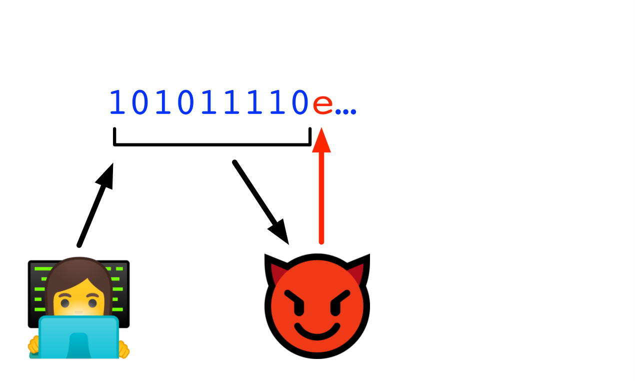

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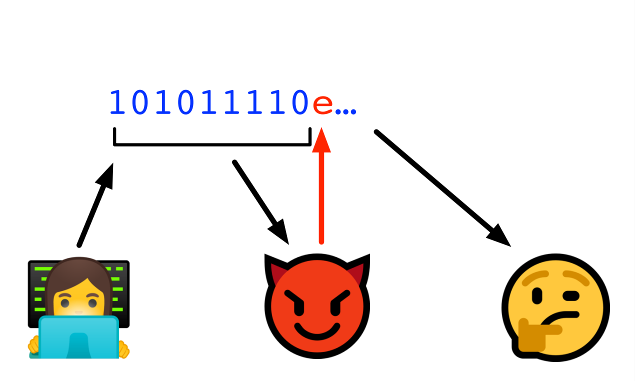

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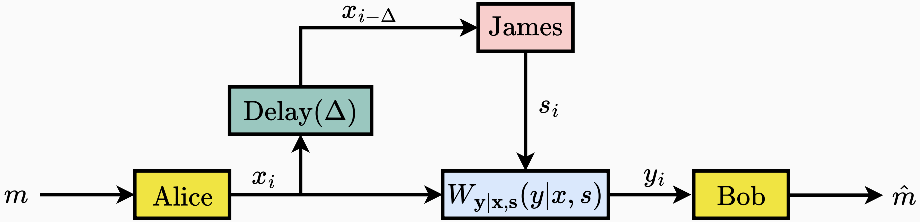
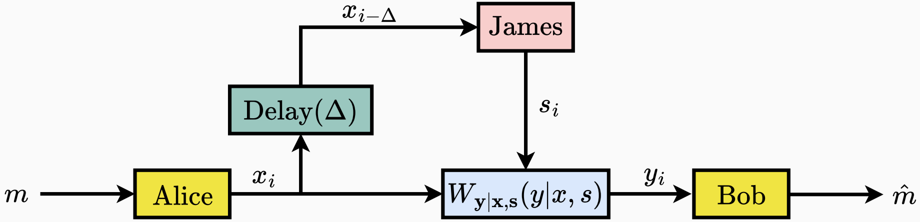


Delayed: James cannot use the full codeword in his strategy

The impact of delay in the erasure setting



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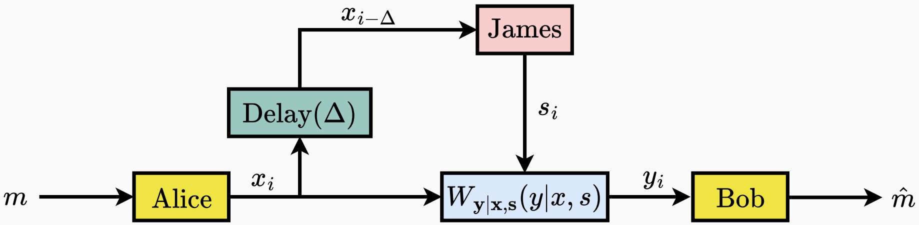

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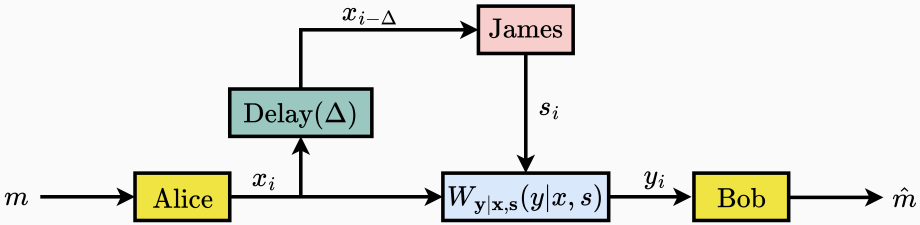

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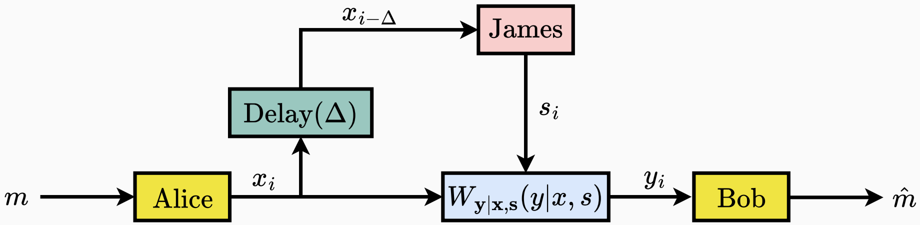

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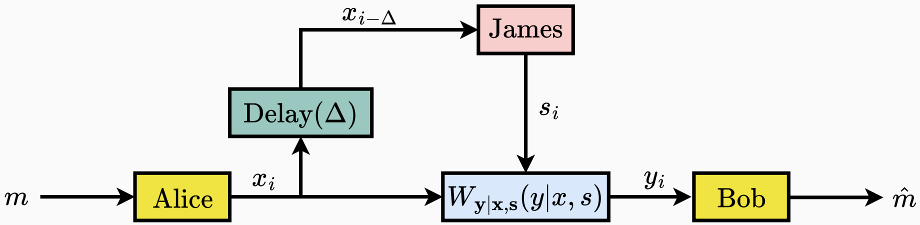

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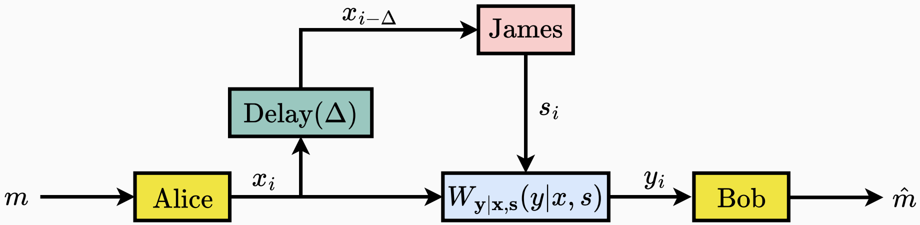

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Knowing the current input gives James a lot of power!

Intermediate model 2: Myopic adversarial models

Myopic: James gets a noisy view of the transmitted codeword.

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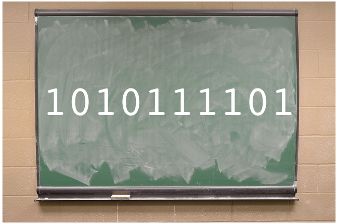

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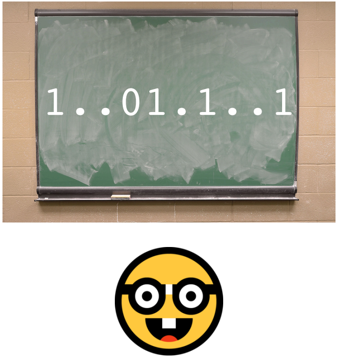

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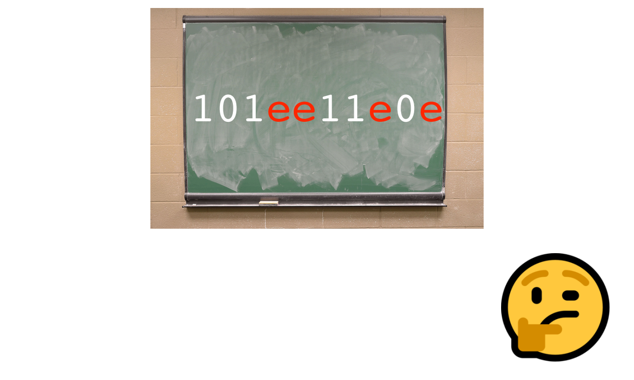

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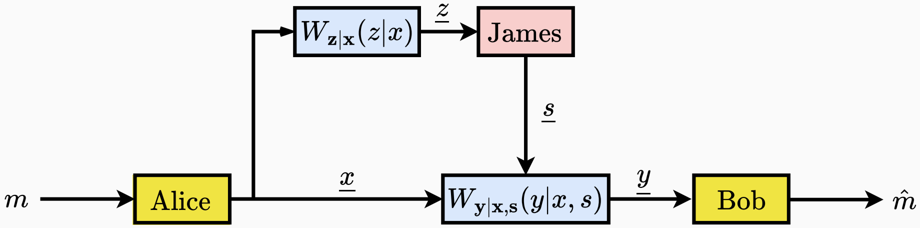

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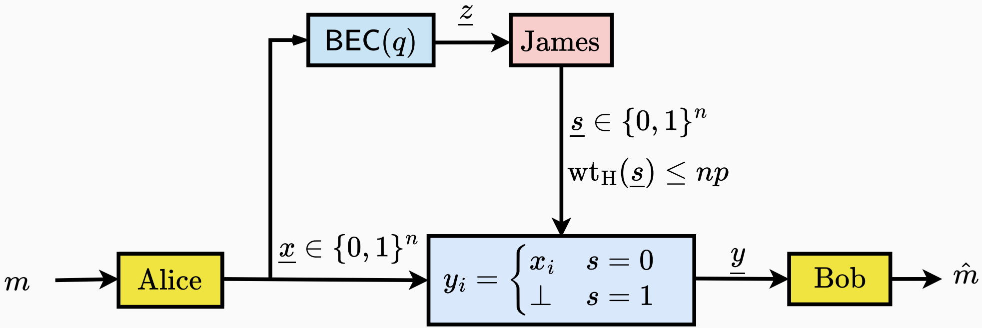

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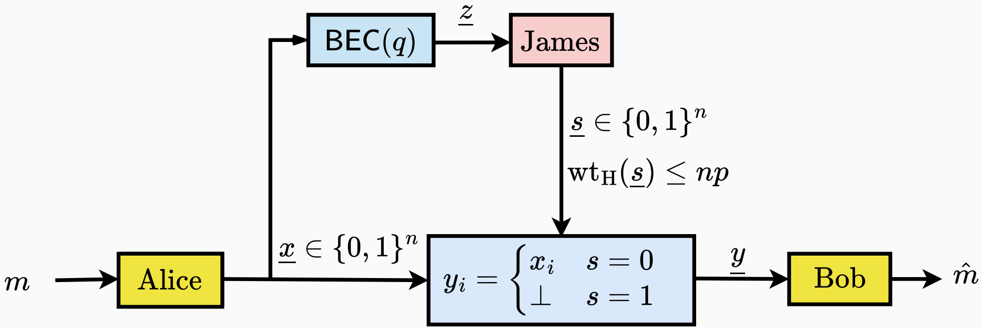

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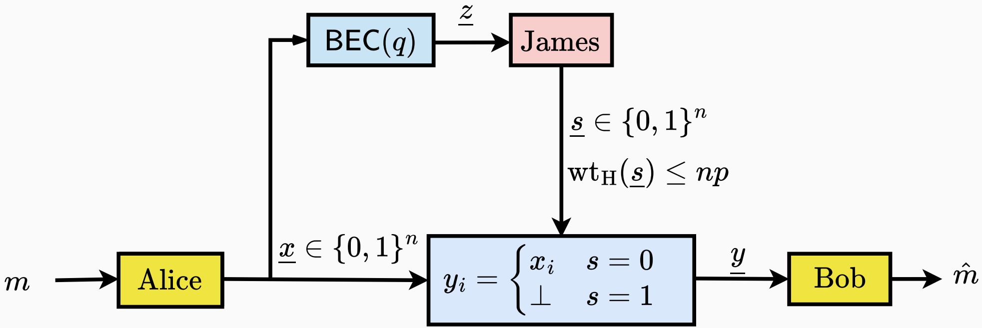
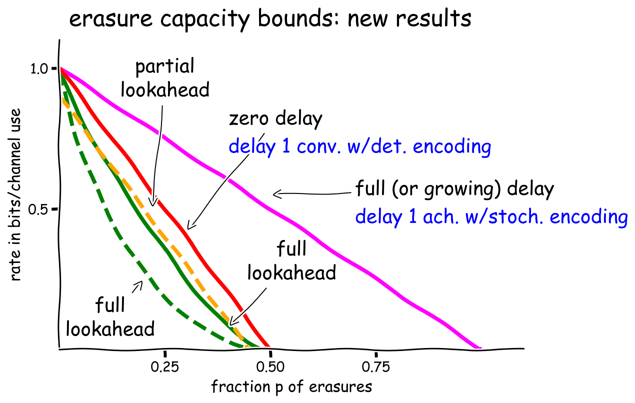


- **Sufficiently myopic:** ($p < q$): capacity $= 1 - p$
- **Otherwise:** ($p > q$): it's more complicated...

Some key ingredients

Ingredient 1: stochastic encoding



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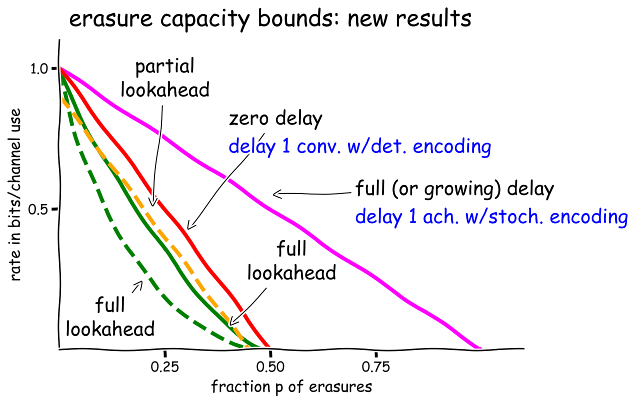

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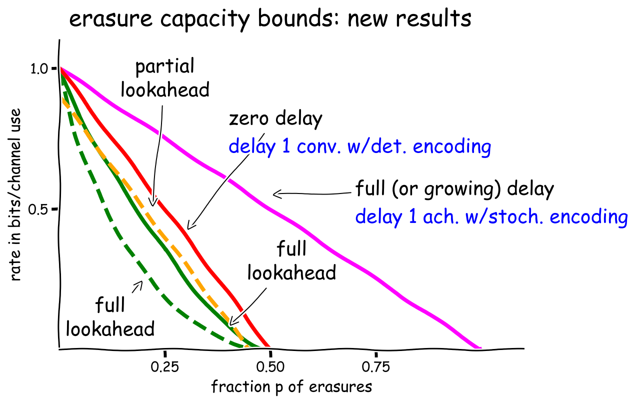

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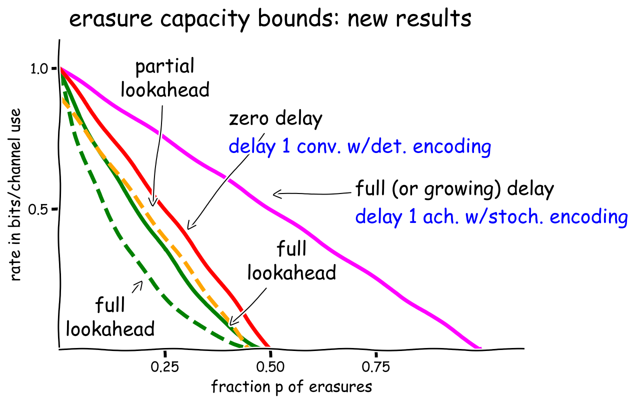

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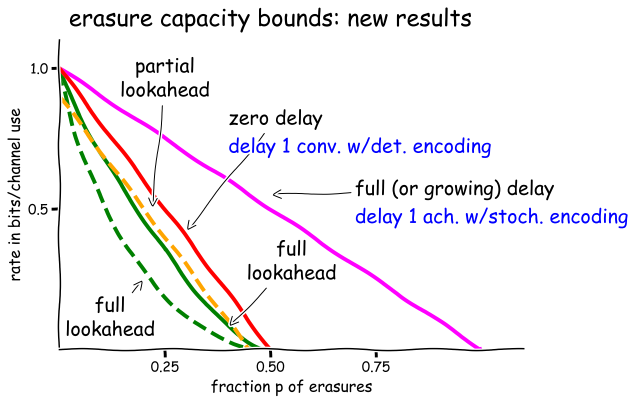

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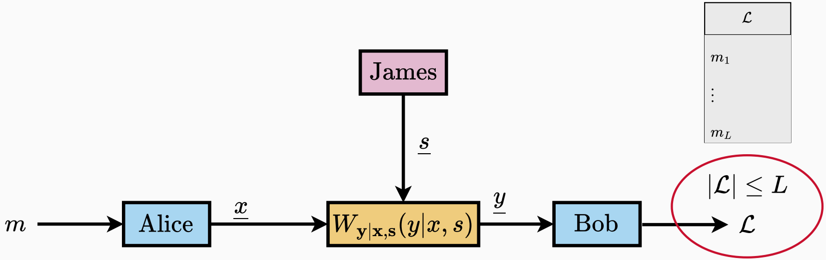
It can be **necessary**: deterministic erasure codes cannot do better than $1 - 2p$ against a James who has a single bit of delay.

Ingredient 2: list decoding



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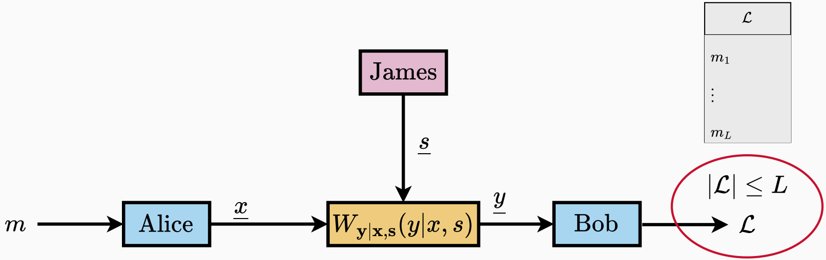

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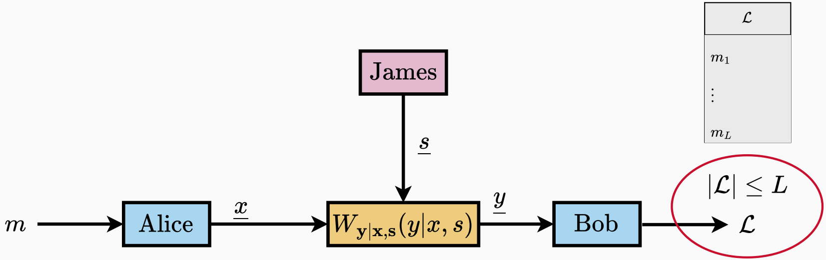

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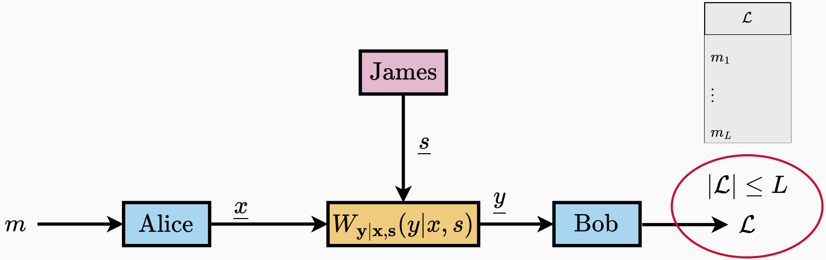

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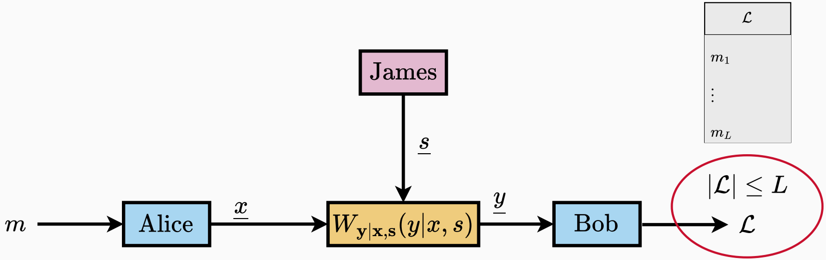

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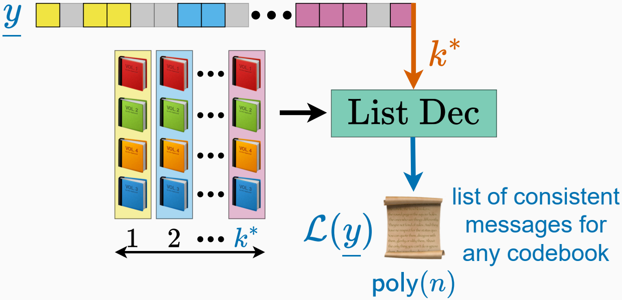
In some cases the list decoding capacity can be **strictly larger**:

$$C_{\text{list}}(L) > C_{\text{obl}}.$$

List decoding can be useful in many ways



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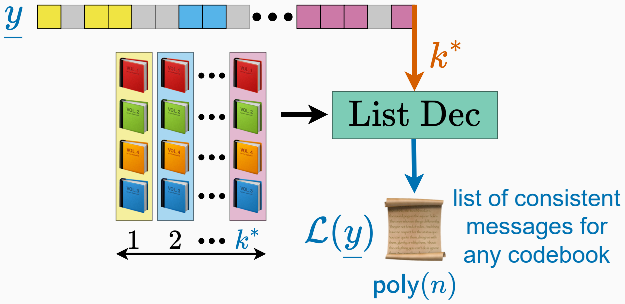

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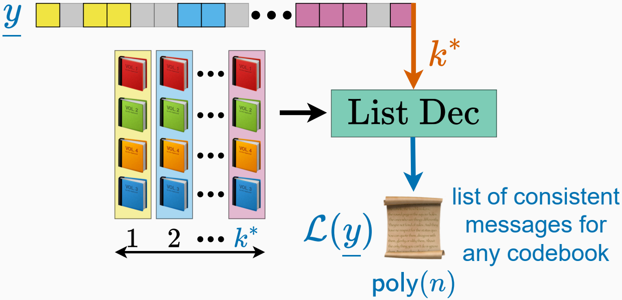

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- Two-stage decoders which sequentially list decode in the first stage.

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- A **self-coupling** is a joint distribution $P_{\mathbf{x},\mathbf{x}'}$ where each marginal is $P_{\mathbf{x}}$.
- A self-coupling is **completely positive** if it is a mixture of independent self-couplings:

$$P_{\mathbf{x},\mathbf{x}'}(x,x') = \sum_{i=1}^{|\mathcal{U}|} P_{\mathbf{u}}(i) P_{\mathbf{x}_i}(x) P_{\mathbf{x}_i}(x').$$

Generalizing the Plotkin bound

Question: can we have a codebook where all codewords have pairwise types that are ρ-far from a CP self-coupling?

$$\|T_{\underline{x},\underline{x}'} - P^{(\mathrm{CP})}_{\mathbf{x},\mathbf{x}'}\|_\infty > \rho \qquad \forall \underline{x}, \underline{x}', P^{(\mathrm{CP})}_{\mathbf{x},\mathbf{x}'} \tag{1}$$

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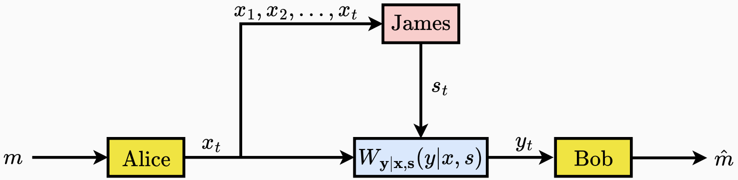
- It turns out that any codes with this property cannot be too large (for large n)!
- Compare this to the Plotkin bound: an upper bound on the size of binary codes with a given distance.
- If our rate is too high, then there will a constant fraction of codeword pairs whose type is close to CP.

Causal adversarial models

Causal adversaries: James can see the current input



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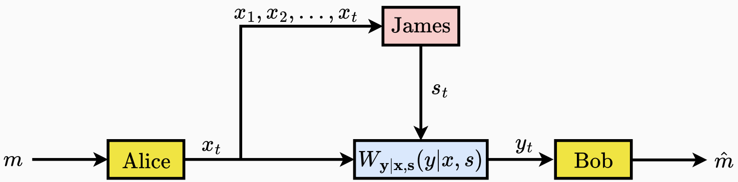

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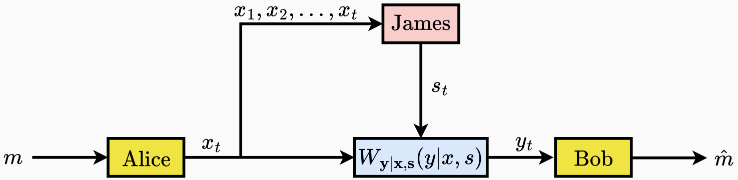

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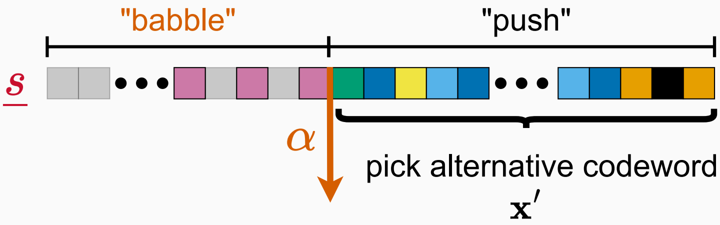
- Spend less power at the beginning to save it up and then push hard in the second half? Bob will get a better initial estimate.
- Spend more power at the beginning in the hope of leading Bob astray? But then the suffix might resolve Bob's uncertainty.

Babble and push: an attack for James



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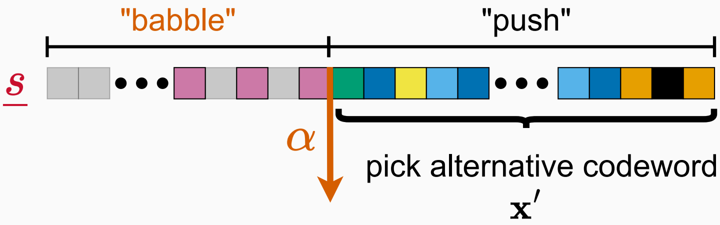

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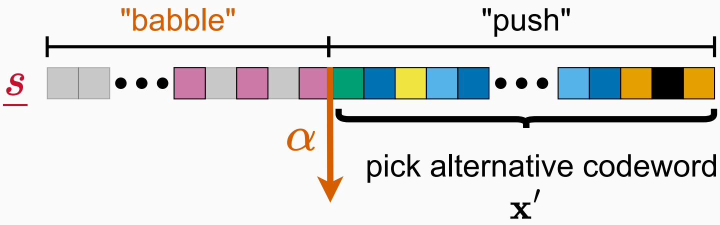

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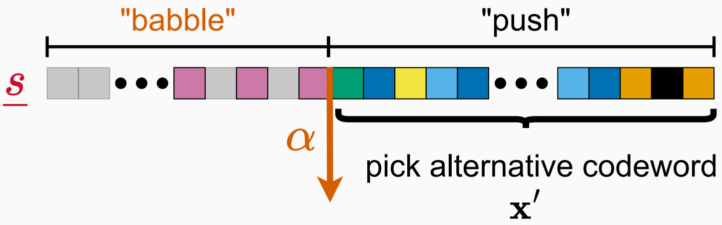

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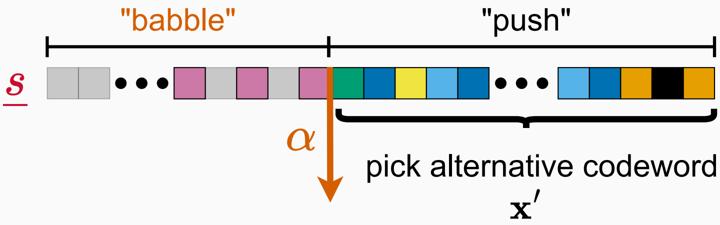

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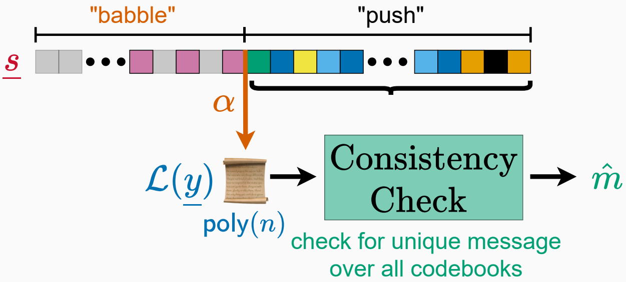
Use the generalized Plotkin bound (plus more) to show this will work.

Achievability



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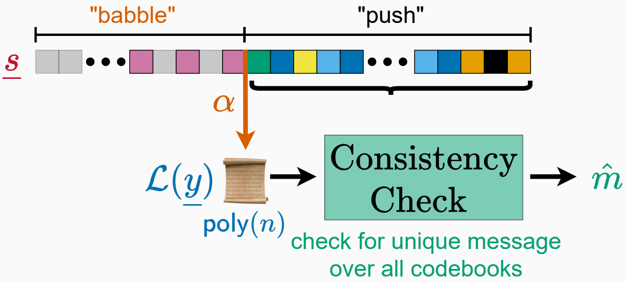

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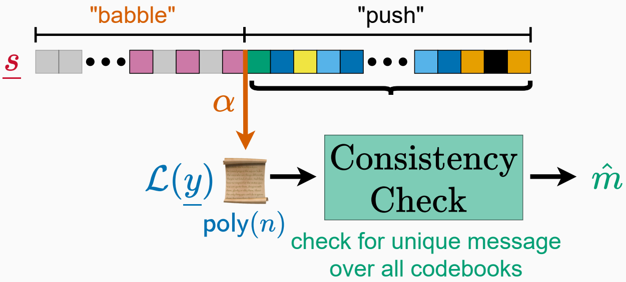

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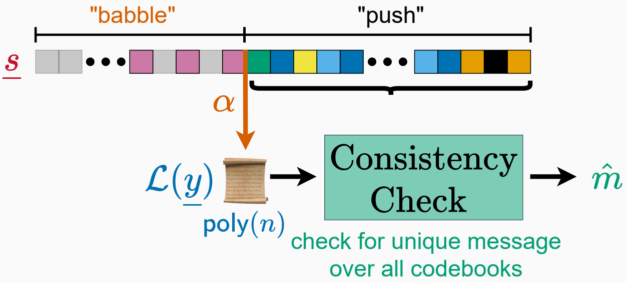

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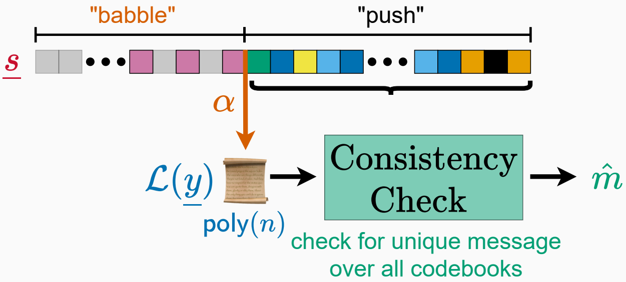

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Basically have to define what "feasible" means in this setting (quite involved).

A multi-letter block characterization

Pros and cons:

A multi-letter block characterization

$$C := \limsup_{K\to\infty} \max_{\substack{P_{\mathbf{x}|\mathbf{u}}\in\Delta(\mathcal{X}|[1:K]) \\ \left[\mathrm{Unif}([K])P_{\mathbf{x}|\mathbf{u}}\right]_{\mathbf{x}}\in\Lambda_{\mathbf{x}}}} \min\Bigg\{ \min_{V_{\mathbf{s}|\mathbf{x},\mathbf{u}}\in\mathcal{F}(P_{\mathbf{x}|\mathbf{u}})} I(P_{\mathbf{x}|\mathbf{u}}, V_{\mathbf{s}|\mathbf{x},\mathbf{u}}),$$

$$\min_{\substack{(\alpha,(V_{\mathbf{s}|\mathbf{x},\mathbf{u}^{\leqslant\alpha}},V_{\mathbf{s}|\mathbf{x},\mathbf{x}',\mathbf{u}^{>\alpha}}))\in\left\{0,\frac{1}{K},\frac{2}{K},\cdots,1\right\}\times\mathcal{F}_{\alpha}(P_{\mathbf{x}|\mathbf{u}}) \\ \forall u\in[\alpha K+1:K],\, V_{\mathbf{s}|\mathbf{x},\mathbf{x}',\mathbf{u}^{>\alpha}=u}\in\mathcal{V}}} I(P_{\mathbf{x}|\mathbf{u}^{\leqslant\alpha}}, V_{\mathbf{s}|\mathbf{x},\mathbf{u}^{\leqslant\alpha}})\Bigg\}.$$

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- ✗ We end up with a multi-letter expression for the capacity.

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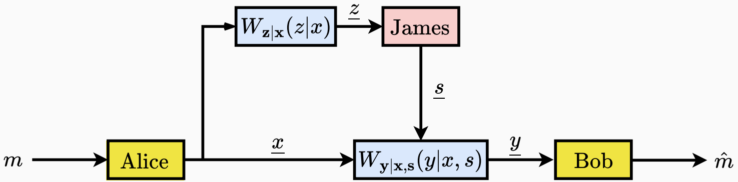
Myopic adversarial models

Myopic adversaries: James sees the whole codeword in noise



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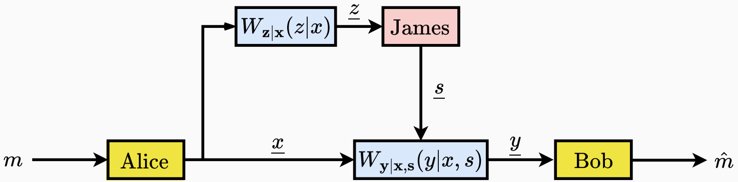

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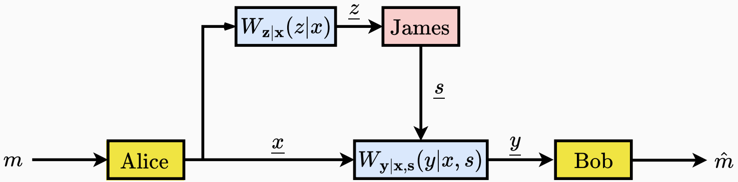

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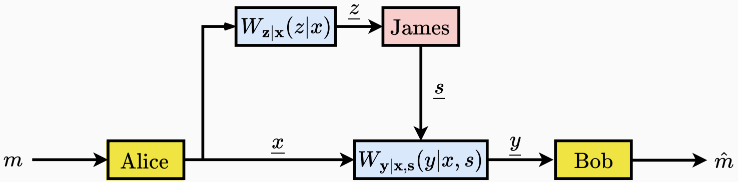

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- By changing $W_{\mathbf{z}|\mathbf{x}}$ we can get the oblivious and omniscient settings.

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Sufficient myopia and achievability

James can create an “effective DMC”

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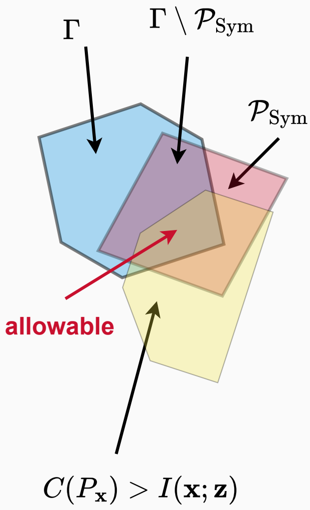

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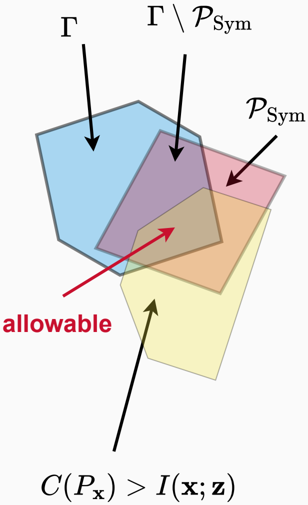

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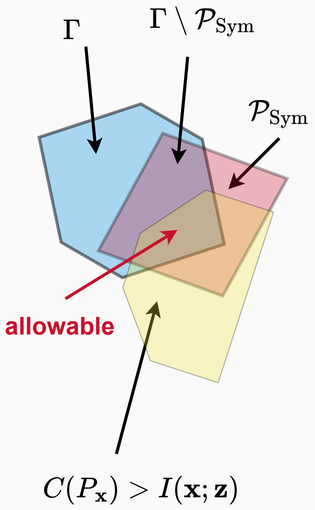
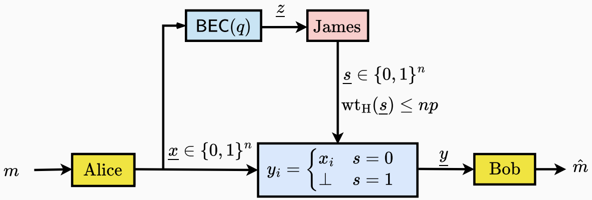
If $I(\mathbf{z};\mathbf{x}) < C(P_{\mathbf{x}})$ we say James is **sufficiently myopic**. In that case we can achieve any rate

$$R < \max_{P_{\mathbf{x}} \in \Gamma \setminus \mathcal{P}_{\text{Sym}}} C(P_{\mathbf{x}}).$$



Myopic adversaries in the erasure setting



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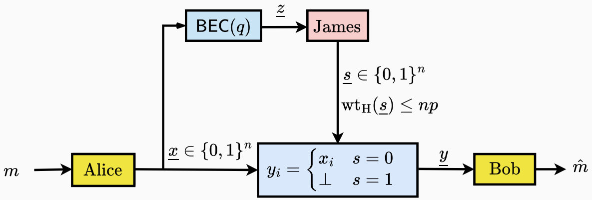

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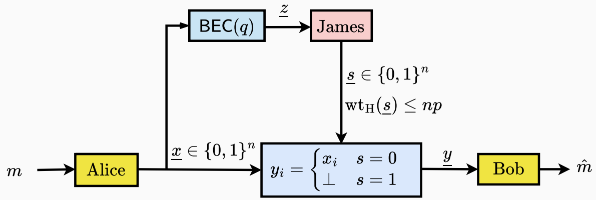

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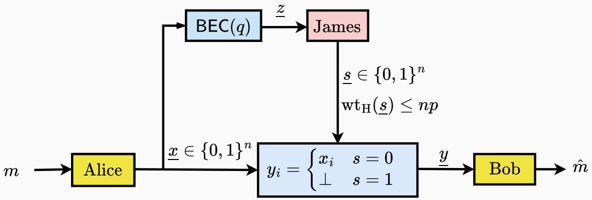

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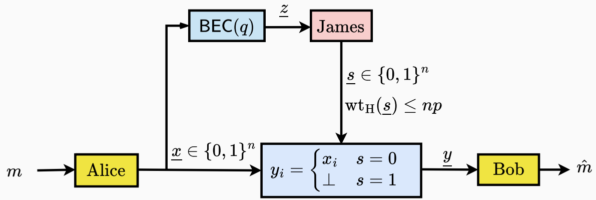

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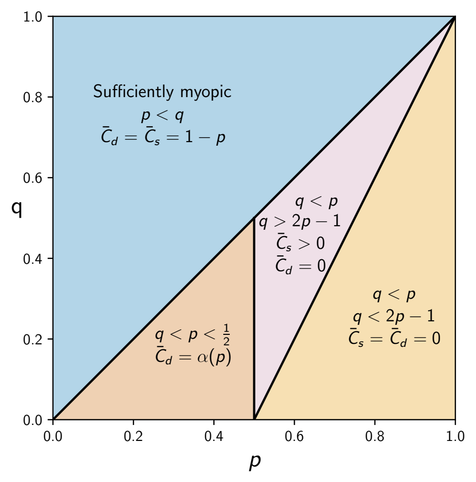

Computationally efficient codes for causal adversaries

Main questions we address

Can we design **efficient codes for causal and myopic models?**

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  $\longrightarrow$ use a **library of linear codebooks**.
- **common randomness** is unrealistic.
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- **minimum distance coding** is not efficient in general.
  $\longrightarrow$ use **list decoding** to permit **efficient decoding**.

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To get **polynomial complexity**, use

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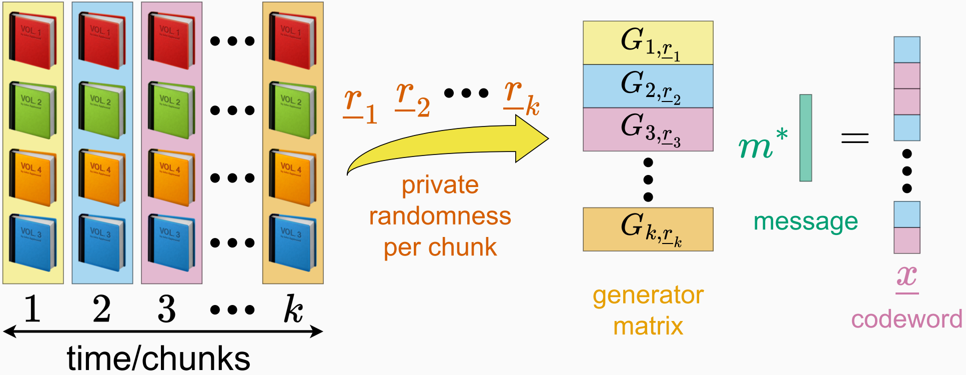
There are different types of complexity we would like to control:

- **Design**: how many bits do we need to generate the code?
- **Storage**: how many bits do we need to store the code?
- **Encoding**: how many operations are needed to encode a message?
- **Decoding**: how many operations are needed to decode the message?

Main results

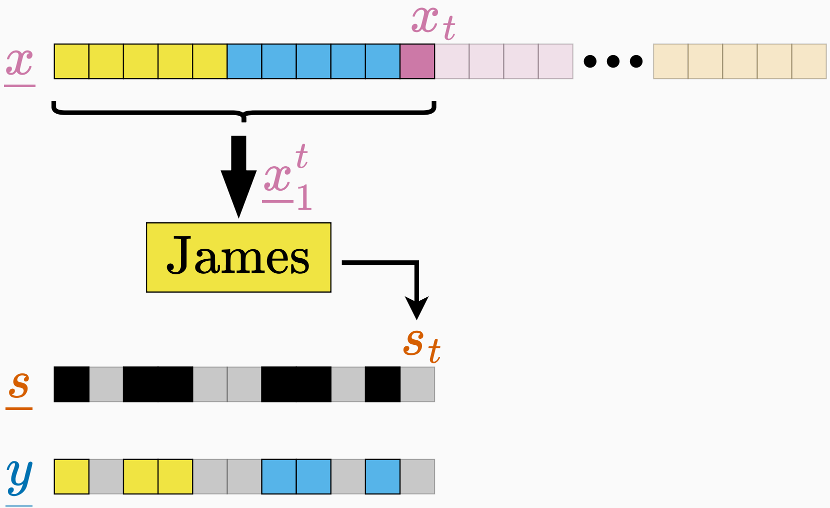
Model **rate**	**Randomness**	**Enc/Storage**	**Decoding**	$\mathbf{P}_{\text{error}}$
Myopic $p < q$ $\mathbf{1-p-\epsilon}$	$\lambda_{SM}\log(n)$	$O(n^{2+\lambda_{SM}})$	$O(n^{3+\lambda_{SM}})$	$O(n^{-\lambda_{SM}})$
Myopic $q < p$ **small rate**	$O(n\log\log n)$	$O(n^2\log\log n)$	$O(n^3\log\log n)$	$O(n^{-4/5})$
Causal $\mathbf{1-2p-\epsilon}$	$O\left(\frac{\gamma\log n}{\epsilon}\right)$	$O(n^3\log\log n)$	$O(n^{32/\epsilon})$	$O(n^{-(\gamma-1)})$

Encode splits block into a constant $k = \lceil \frac{n}{\epsilon} \rceil$ chunks

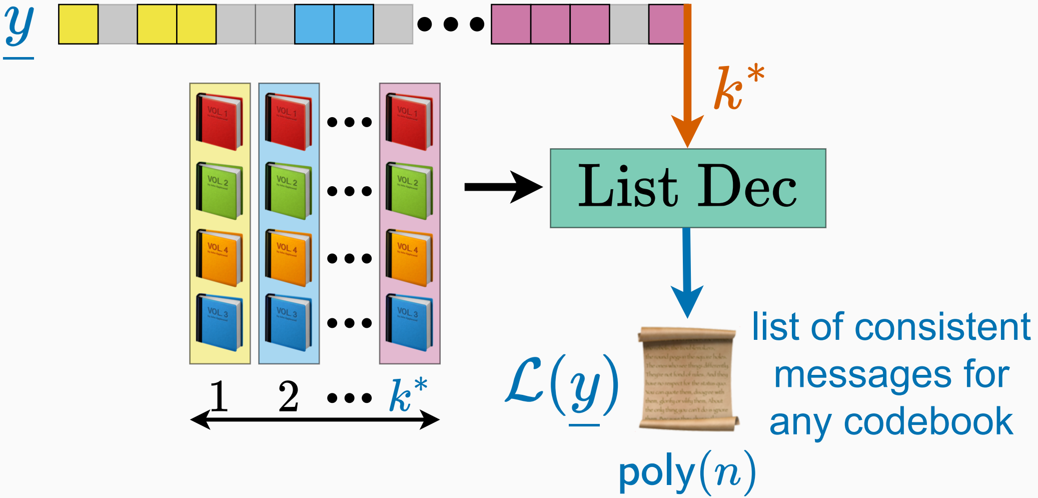


Generate a **library of linear codebooks** independently for each chunk.

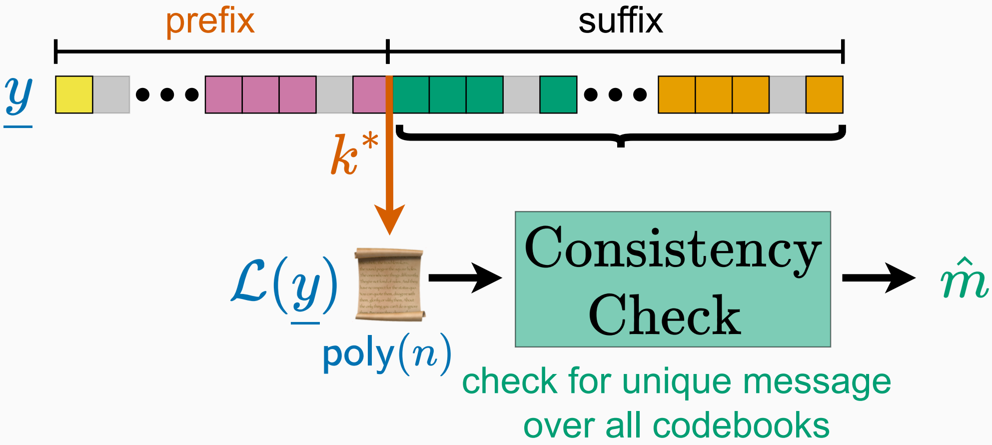
James can erase with causal information only



Bob decodes to a polynomial list



Bob uses suffix to disambiguate the list

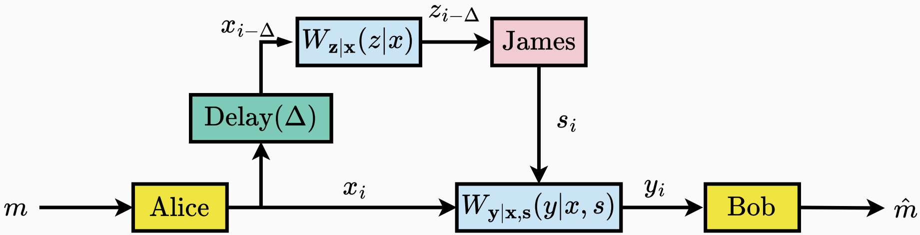


Why does this work?

1. Bob can **track James's erasure budget**.
2. List decoding creates **a smaller set of messages** to check for consistency.
3. James has a choice to **make the list larger** (erase more earlier, less later) or **conserve his budget** (erase less earlier, more later).
4. **Poor James, he can't win.**

Looking forward

Other intermediate models



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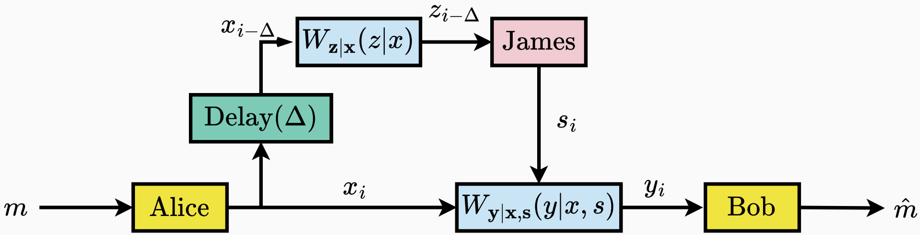

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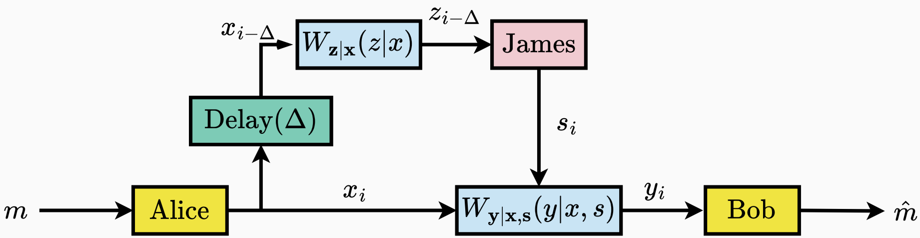

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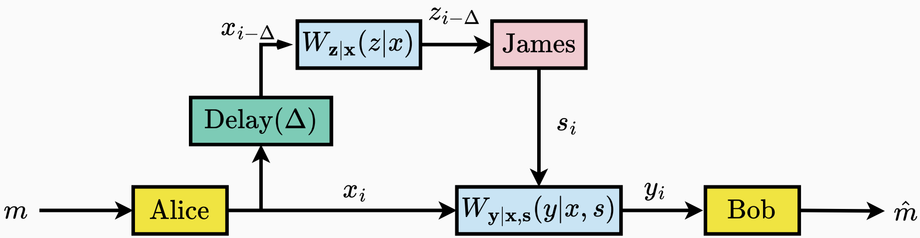

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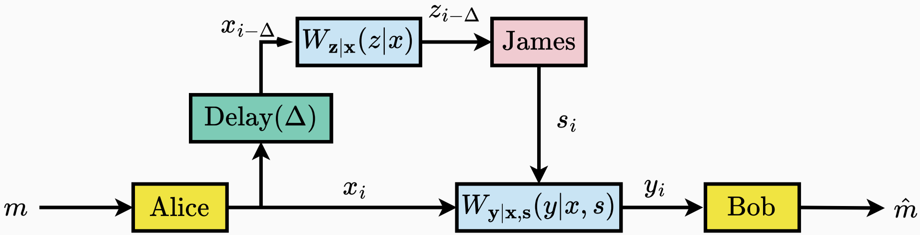

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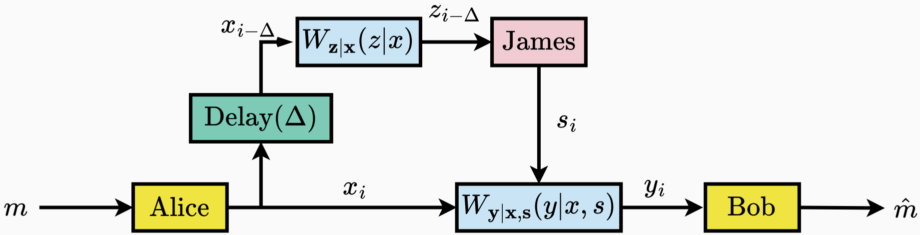

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Each model will reveal something about what the **worst-case channel** looks like.

And for the theory folks...

Understanding AVCs has lots of connections (perhaps less well described here) to many interesting areas:

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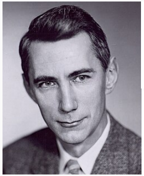
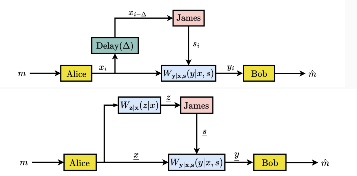
- other fun combinatorial problems

A final recap and takeaways





Proposed **arbitrarily varying channels** to explore the difference between average and worst-case channels.

- When James has to act causally, the capacity depends crucially on **what he knows about the current input**.
- When James has to act myopically, it depends on whether he **"decode"** or not: this creates many connections with the wiretap channel.

For emerging networked systems random noise models may be **too optimistic** and completely adversarial models may be **too pessimistic**. Strategies like **stochastic encoding** and **list decoding** can help!

Thank you!