Communication against restricted adversaries: between Shannon and Hamming

University of Sydney

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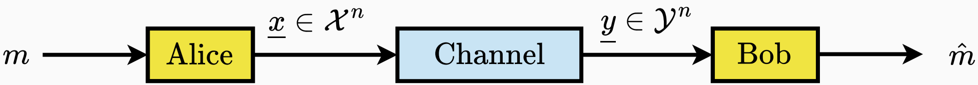
07 August 2025

Reliable communication, revisited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Channel
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Bob
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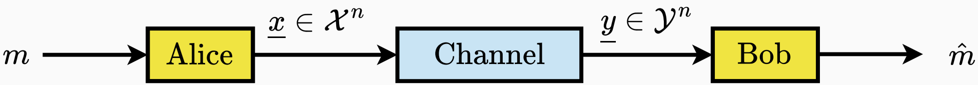
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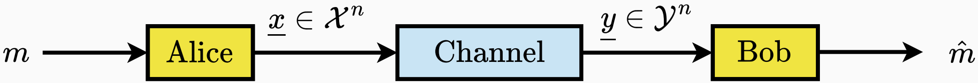
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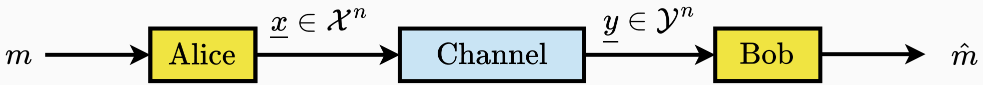

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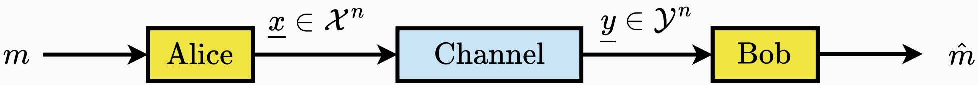
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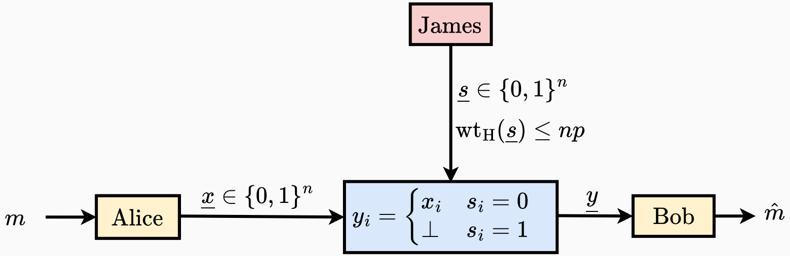
Let's zoom in on binary channels with erasures.

Binary input channels with erasures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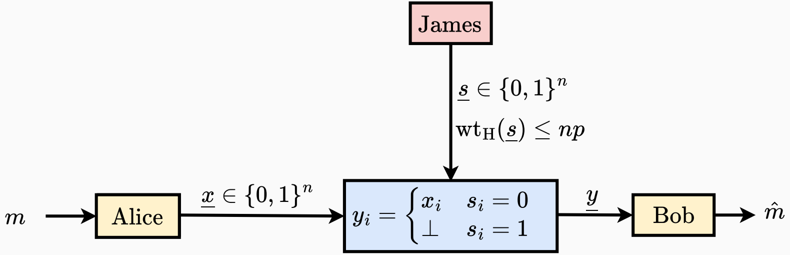
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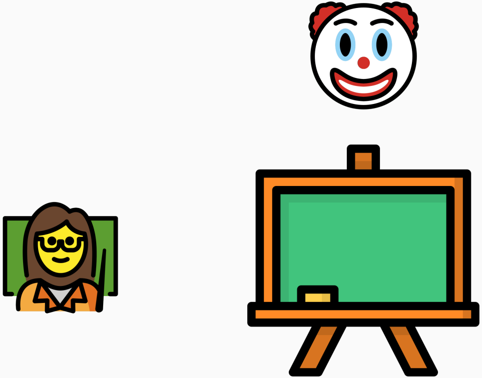
How is $\underline{s}$ chosen?

Shannon theory: the channel is *random*



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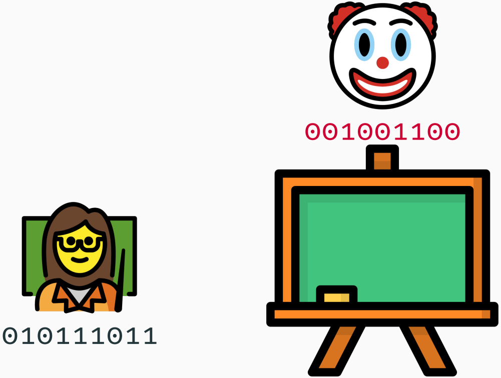

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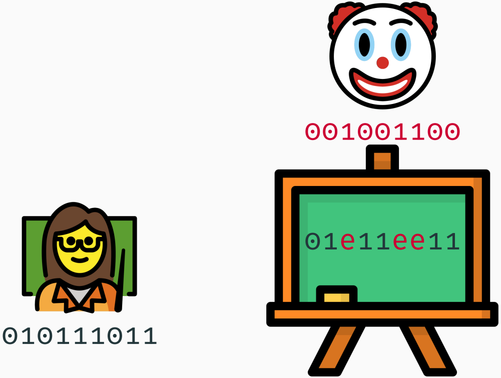

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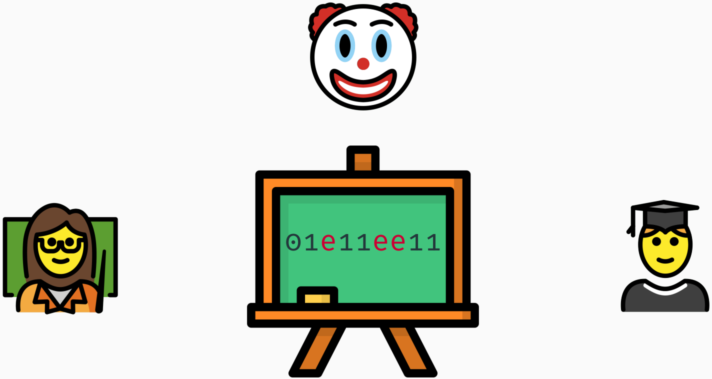

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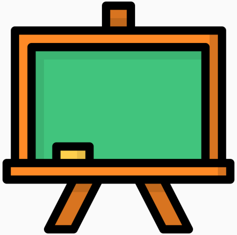
Coding theory (“Hamming”): the erasures are random



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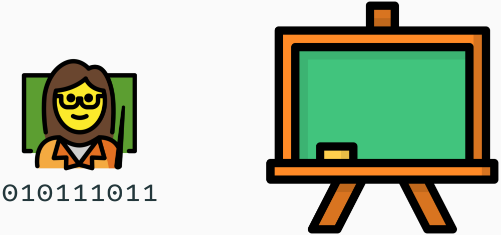

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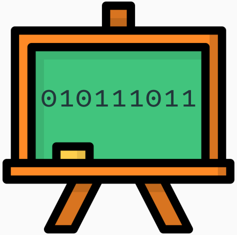

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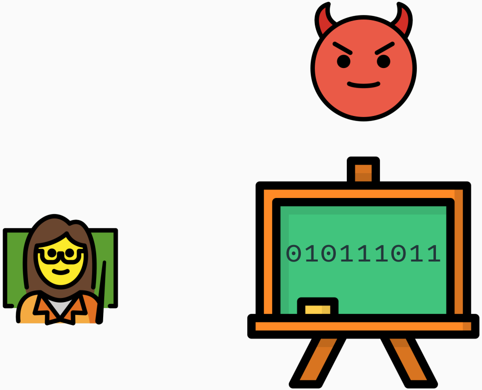

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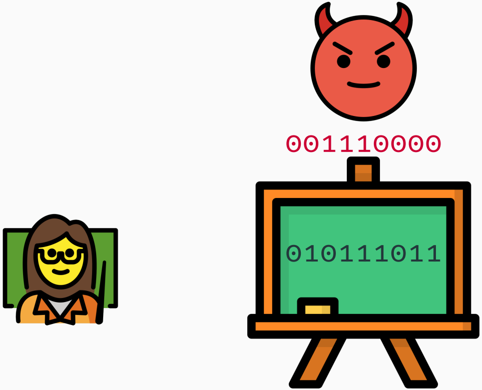

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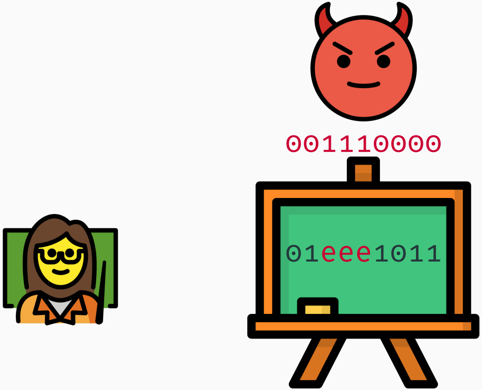

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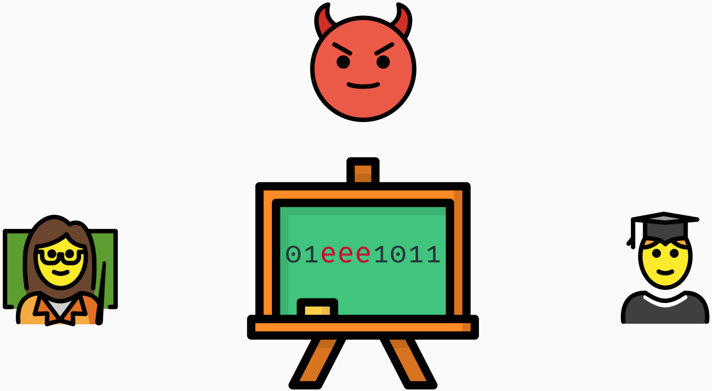

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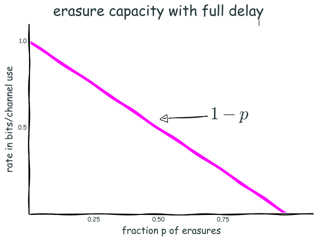
The erasure channel: adversarial vs. random



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$$C = 1 - p.$$

There are many different ways to achieve this rate.

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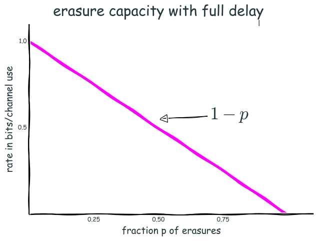

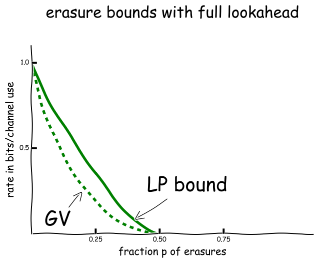

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$$C < 1 - 2p.$$

Lower bound: Gilbert-Varshamov (random) codes.

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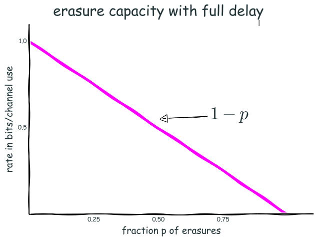

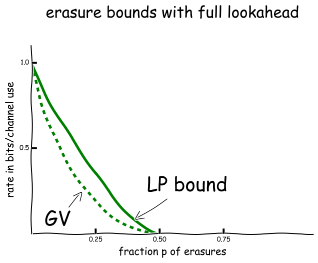

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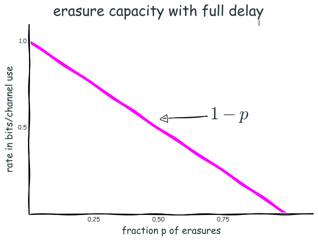

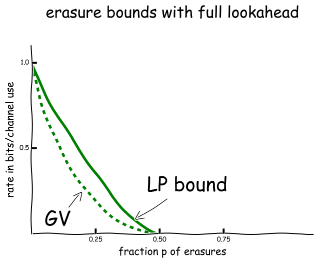

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That's a big gap... where does it come from?

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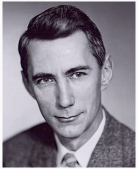

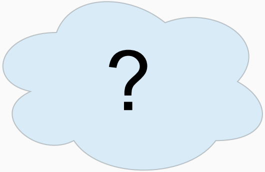

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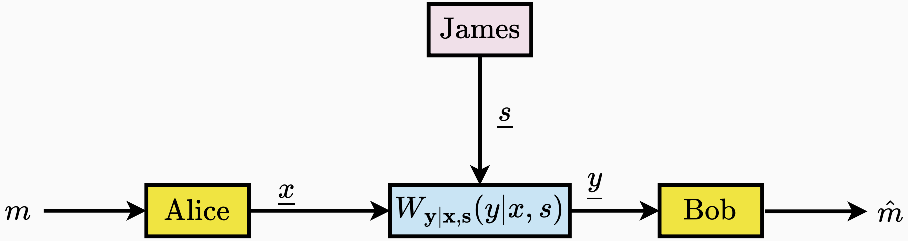
1. Use **arbitrarily varying channels (AVCs)** to develop a **unified framework** for both the Shannon and Hamming models.
2. Explore **intermediate models** to see **what causes the gap**.
3. Discover **coding strategies** to see what **resources are needed** to communicate reliably.

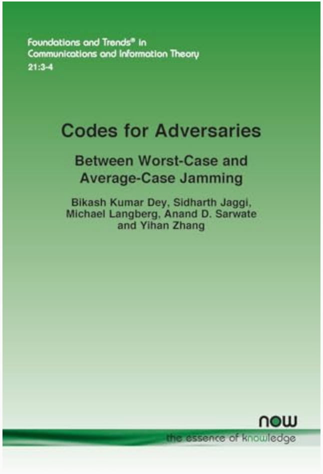
AVCs model channel “noise” as a state variable



In an **adversarial channel model**, **Alice** wants to communicate with **Bob** over a channel whose time-varying state is controlled by an adversarial **jammer** James.

- Alice and James may be **constrained** in how they communicate.
- Capacity depends on **what James knows** about m and $\underline{x}$.

Shameless self-promotion



Foundations and Trends® in
Communications and Information Theory
21:3-4

Codes for Adversaries

Between Worst-Case and
Average-Case Jamming

Bikash Kumar Dey, Sidharth Jaggi,
Michael Langberg, Anand D. Sarwate
and Yihan Zhang

now
the essence of knowledge

We have a monograph (December 2024!) on this topic (on which this talk is based).

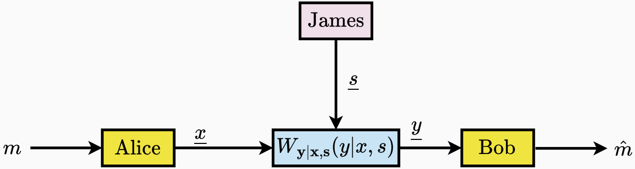
- ✔ Unified treatment of random noise (Shannon-theoretic) and worst-case noise (coding-theoretic).
- ✔ Intermediate models for jammers who can eavesdrop: online and myopic.
- ✔ Examples, open problems, and more!

What’s coming up next

1. Arbitrarily varying channels (AVCs)
2. Some key ingredients
3. Causal adversarial models
4. Myopic adversarial models
5. Computationally efficient codes for causal adversaries
6. Looking forward

Arbitrarily varying channels (AVCs)

The basic channel model



Let $\mathcal{X}$, $\mathcal{S}$, and $\mathcal{Y}$ be discrete alphabets. An AVC is a discrete channel $W_{\mathbf{y}|\mathbf{x},\mathbf{s}}(y|x,s)$ such that

$$W_{\underline{\mathbf{y}}|\underline{\mathbf{x}},\underline{\mathbf{s}}}(\underline{y}|\underline{x},\underline{s}) = \prod_{i=1}^{n} W_{\mathbf{y}|\mathbf{x},\mathbf{s}}(y_i|x_i,s_i)$$

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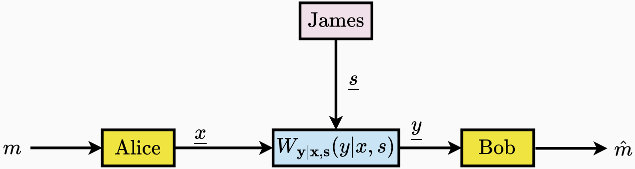

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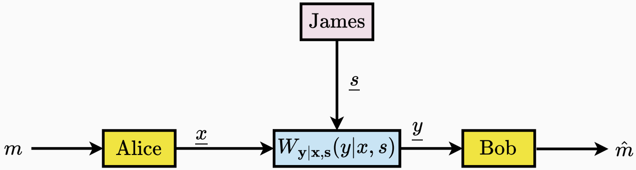

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Examples: For binary channels $\underline{s}$ could be the error erasure pattern.

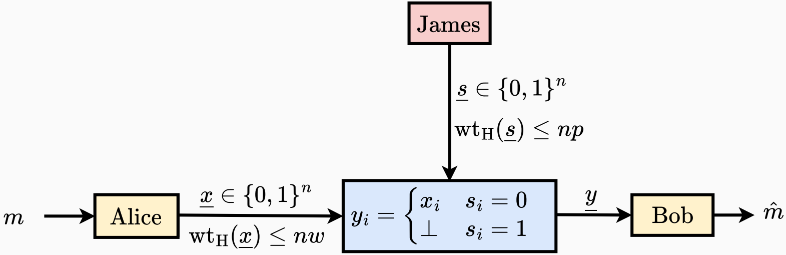
Input and cost constraints for AVCs

We impose that the types $T_{\underline{x}}$ and $T_{\underline{s}}$ of the codeword $\underline{x}$ and the state $\underline{s}$ lie be in convex subsets of the probability simplices $\Delta(\mathcal{X})$ and $\Delta(\mathcal{S})$:

$$T_{\underline{x}} \in \Gamma \subseteq \Delta(\mathcal{X})$$

$$T_{\underline{s}} \in \Lambda \subseteq \Delta(\mathcal{S})$$

Example: For binary channels $\underline{x}$ and $\underline{s}$ have bounded Hamming weight.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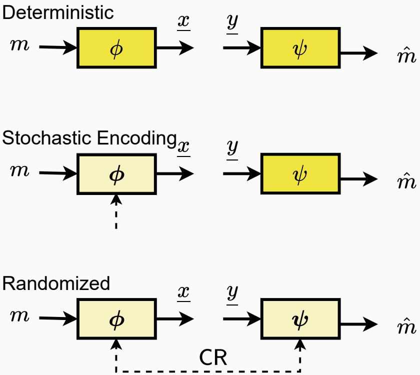
Defining codes and input constraints



An (n, M, Γ) code is

$$\phi \colon [M] \to \mathcal{X}^n \qquad \text{(encoder)}$$
$$\psi \colon \mathcal{Y}^n \to [M] \qquad \text{(decoder)}$$

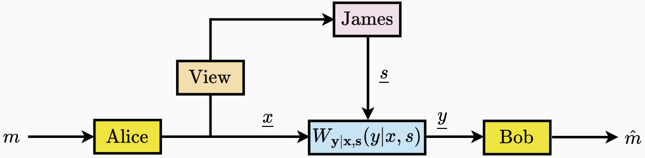
such that

$$T_{\phi(m)} \in \Gamma$$

The rate is $R = \frac{1}{n} \log_2(M)$.

A **randomized code** lets Alice and Bob choose their code in secret. If Alice and Bob do not share common randomness, Alice can still use **stochastic encoding**.

What James knows: Shannon, Hamming, and in between



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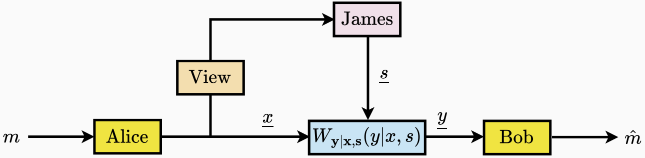

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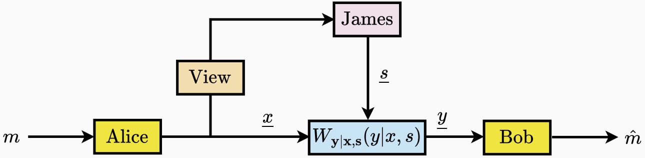

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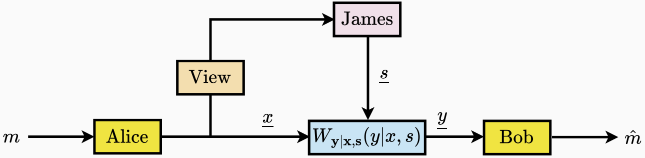

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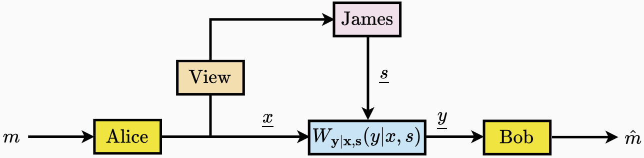

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These constrain the set of **strategies** James can use.

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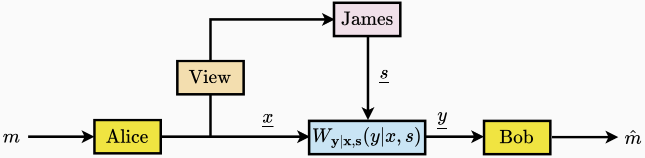

James wants to choose $\underline{s}$ to maximize the probability of error for **Bob**. What James can do depends on what he knows:

- The message: target small **maximal (over messages) error**.
- The codeword (fully or partially).
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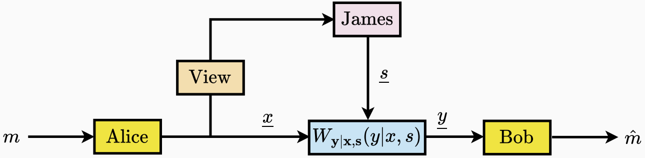

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- **Oblivious** (Shannon): the message only.
- **Omniscient** (Hamming): the message and the codeword.

Maximal error and capacity

Maximal and average error:

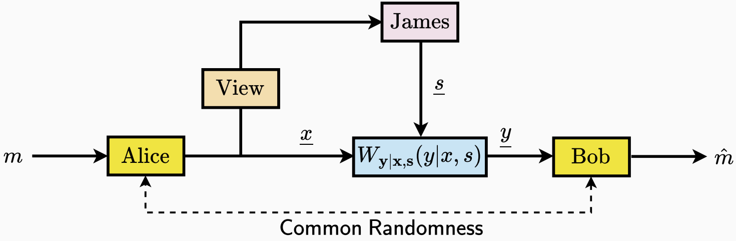
$$P_{\mathrm{err}}(m,\phi,\psi) = \max_{\text{jamming strategies}} \sum_{\mathbf{x}\in\mathcal{X}^n} \mathbb{P}\left(\psi(\mathbf{y}) \neq m \mid \mathbf{x}\right) \mathbb{P}_{\phi}\left(\phi(m) = \mathbf{x}\right)$$

A rate R is achievable if for any $\epsilon > 0$ there exists an infinite sequence of rate R codes such that $P_{\mathrm{err}}(m,\phi,\psi) < \epsilon$ for all m.

The capacities C_{obl} and C_{omni} for oblivious and omniscient cases satisfy:

$$(\textit{Hamming}) \qquad C_{\mathrm{omni}} \qquad \leq \qquad C_{\mathrm{obl}} \qquad (\textit{Shannon})$$

Common randomness makes the problem easier



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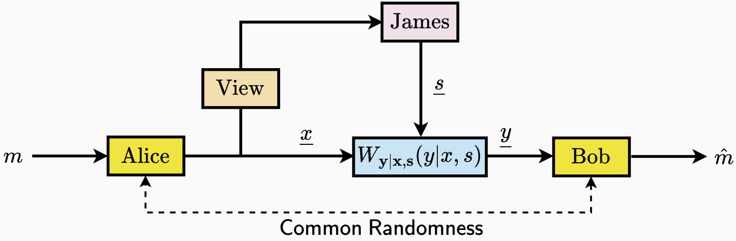

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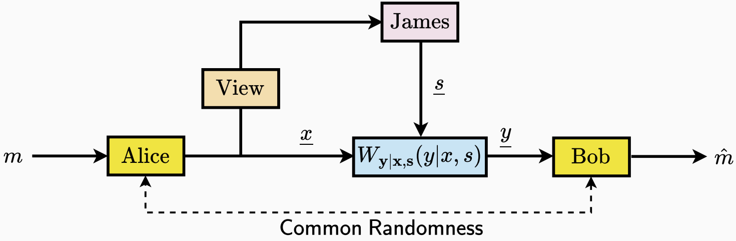

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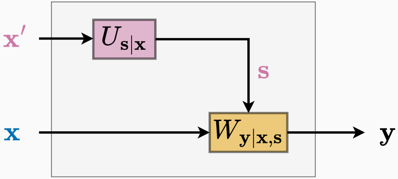
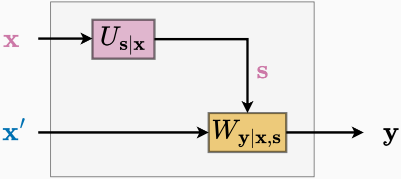
Deterministic codes and ECN Symmetrizability



$\mathbf{x}'$
$U_{\mathbf{s}|\mathbf{x}}$
$\mathbf{s}$
$\mathbf{x}$
$W_{\mathbf{y}|\mathbf{x},\mathbf{s}}$
$\mathbf{y}$



$\mathbf{x}$
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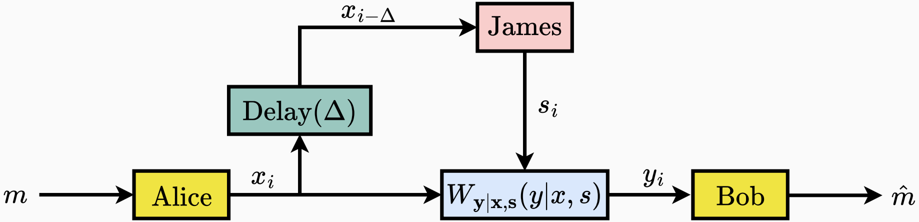
An AVC is **Ericson-Csiszár-Narayan (ECN) symmetrizable** if James can spoof Alice's codeword. That is, for all $(\mathbf{y}, \mathbf{x}, \mathbf{x}')$, we have

$$\sum_{s} U_{\mathbf{s}|\mathbf{x}'} W_{\mathbf{y}|\mathbf{x},\mathbf{s}} = \sum_{s} U_{\mathbf{s}|\mathbf{x}} W_{\mathbf{y}|\mathbf{x}',\mathbf{s}}.$$

Without common randomness, the capacity of a symmetrizable AVC $C_{\mathrm{obl}} = 0$.

Intermediate model 1: delayed information for James



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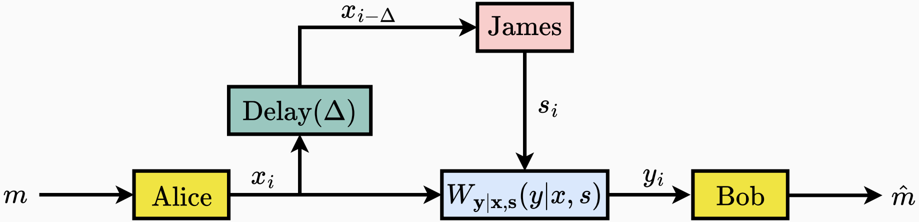

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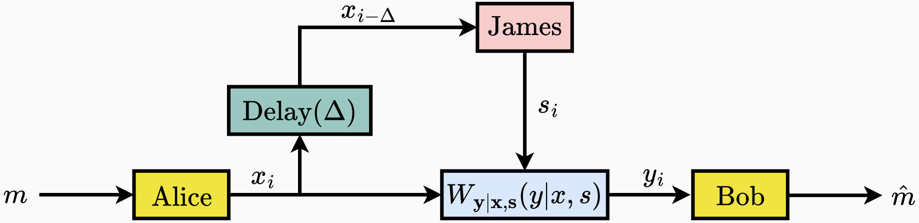

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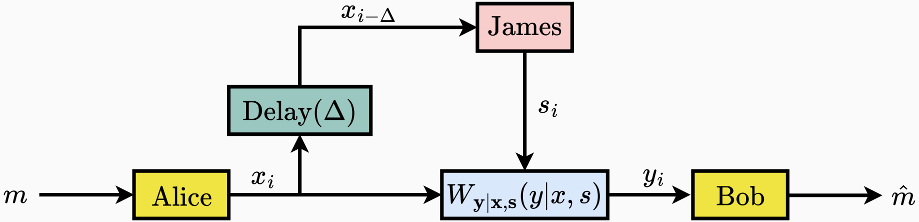

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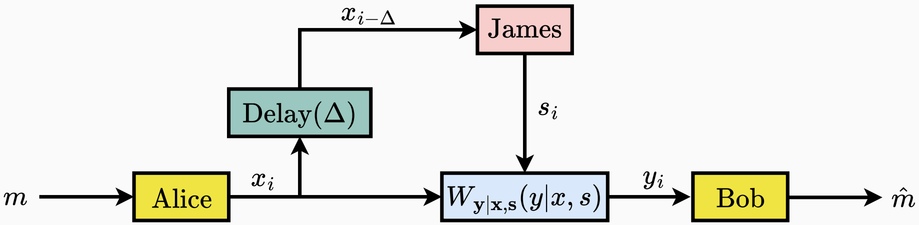

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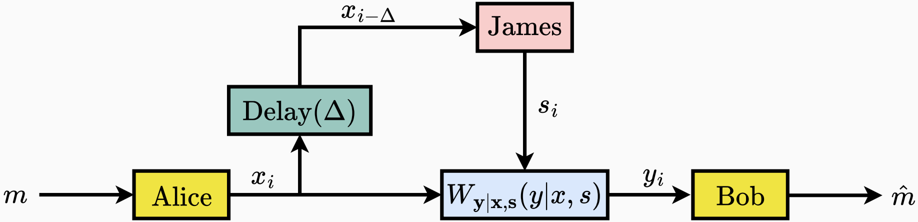

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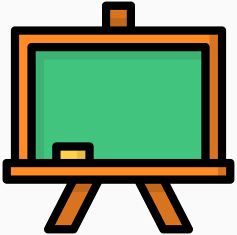
Knowing the current input gives James a lot of power!

Intermediate model 2: Myopic adversarial models



Myopic: James gets a noisy view of the transmitted codeword.

Intermediate model 2: Myopic adversarial models

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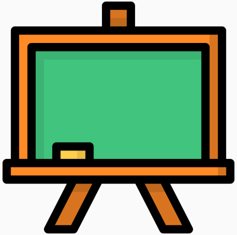

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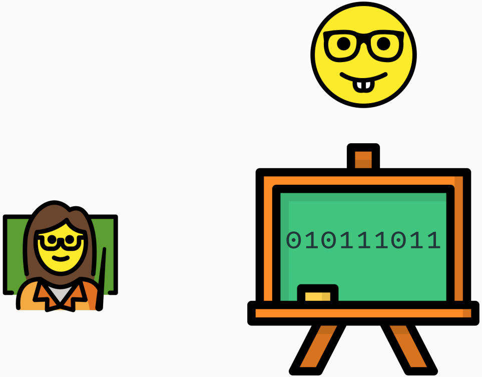

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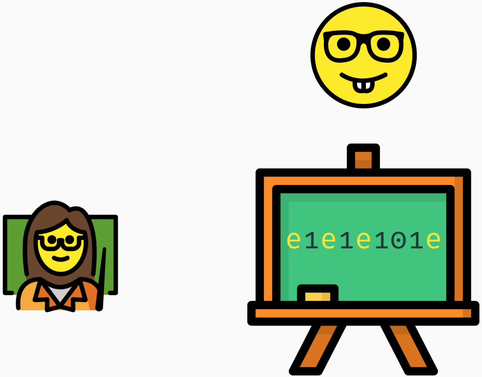

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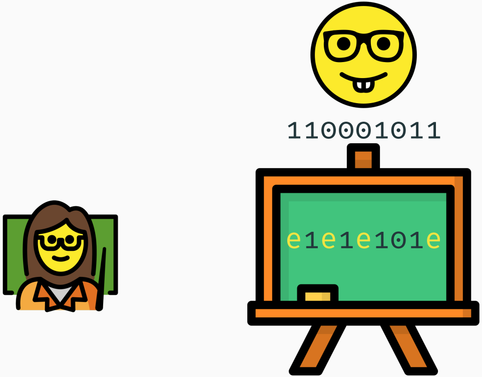

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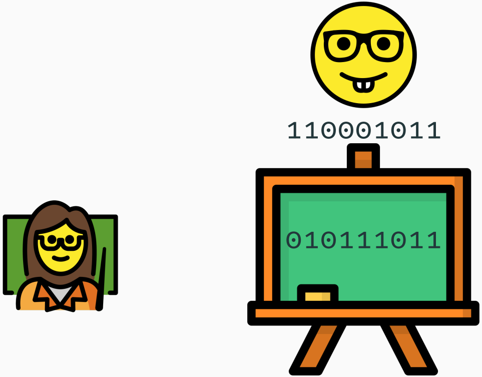

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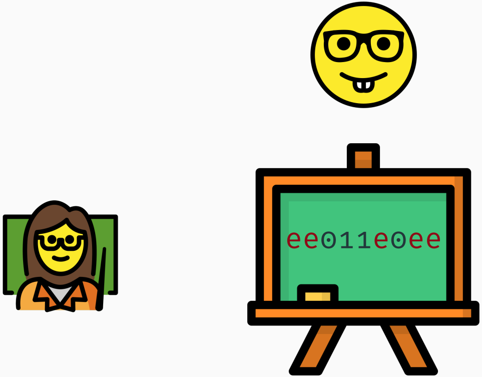

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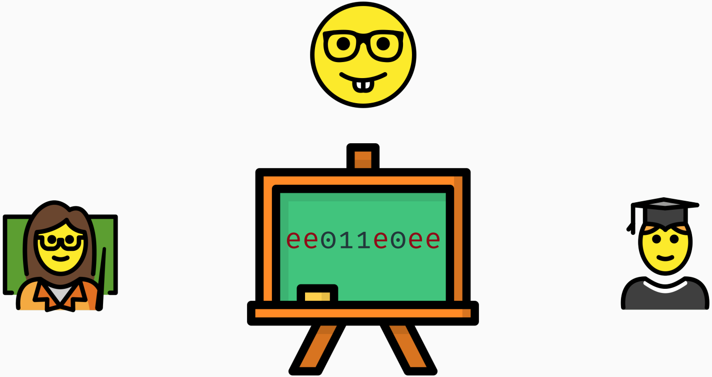

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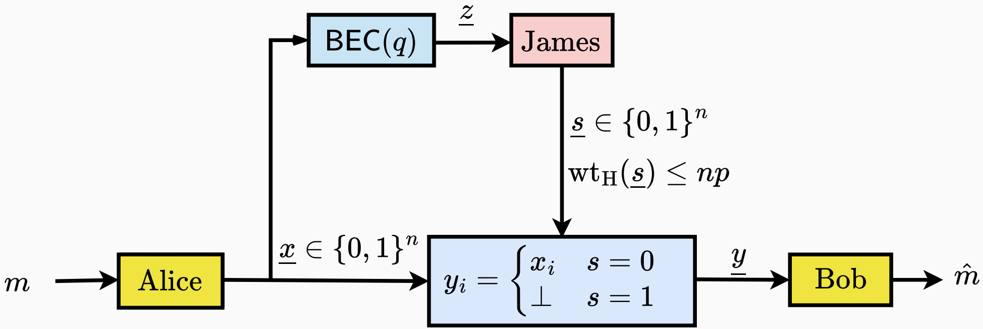
The impact of myopia in the erasure setting


BEC(q)
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James
$\underline{s} \in \{0,1\}^n$
$\mathrm{wt_H}(\underline{s}) \leq np$
m
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$y_i = \begin{cases} x_i & s = 0 \\ \perp & s = 1 \end{cases}$
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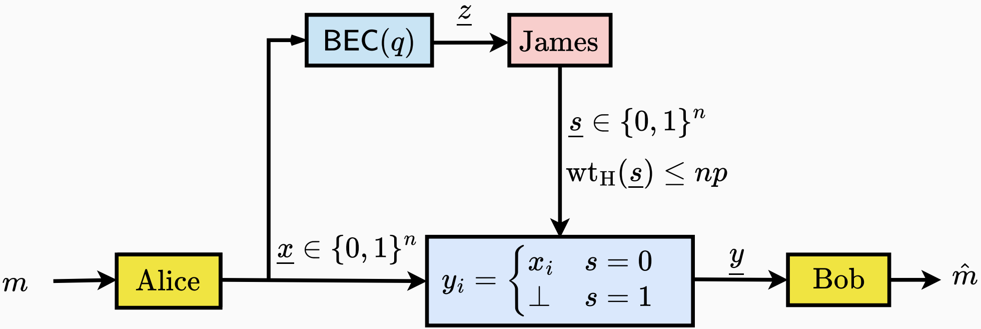

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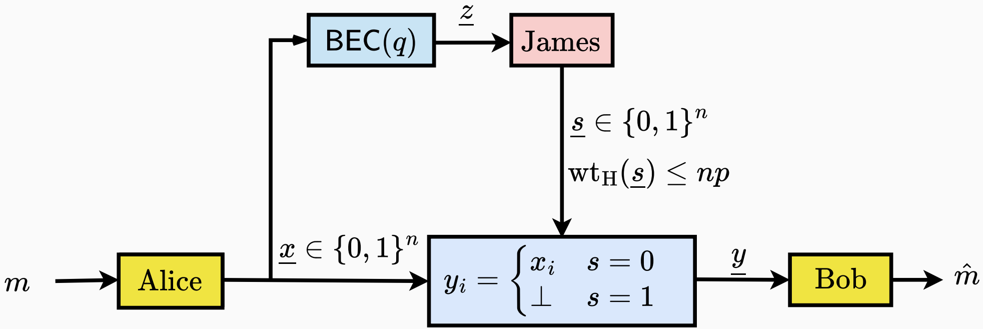

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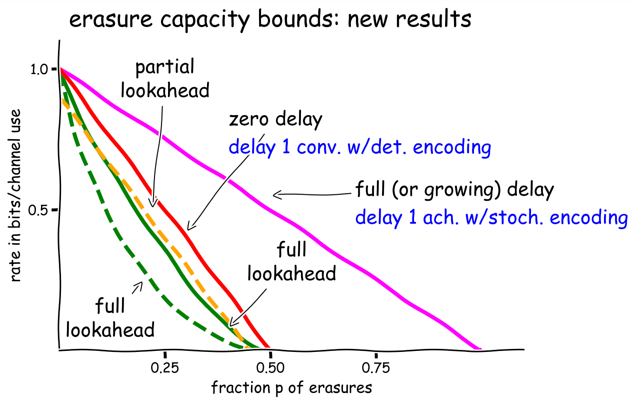
- **Sufficiently myopic:** ($p < q$): capacity $= 1 - p$
- **Otherwise:** ($p > q$): it's more complicated...

Some key ingredients

Ingredient 1: stochastic encoding



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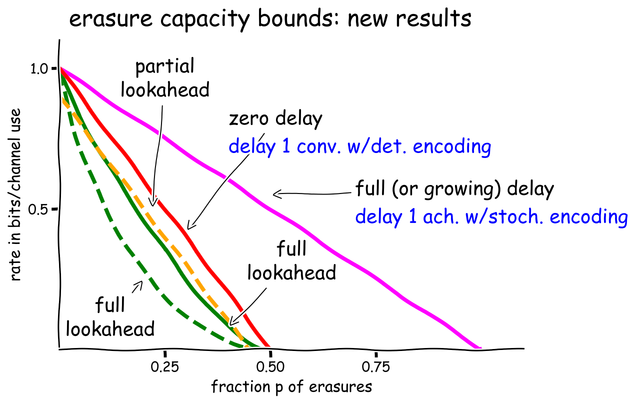

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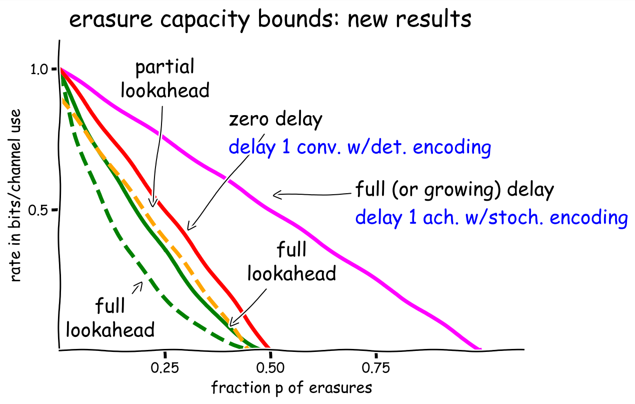

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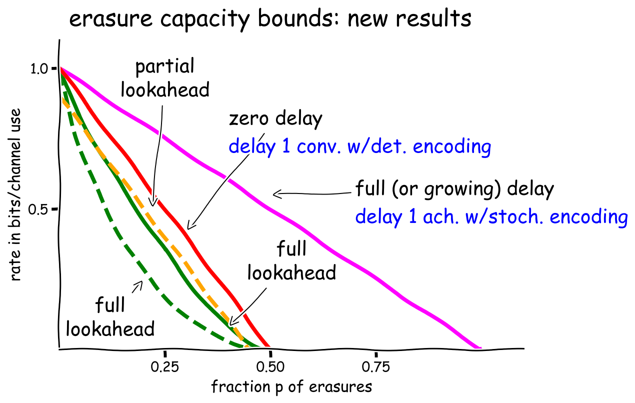

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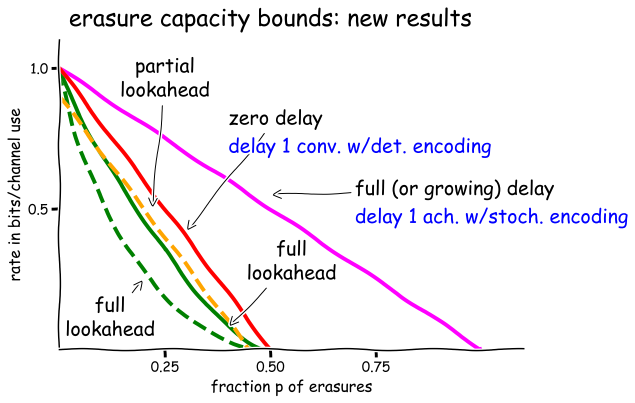

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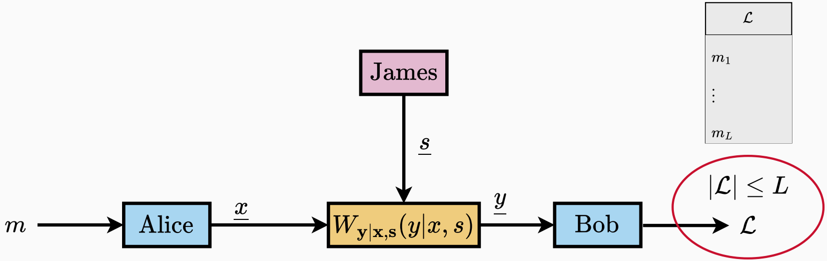
It can be **necessary**: deterministic erasure codes cannot do better than $1 - 2p$ against a James who has a single bit of delay.

Ingredient 2: list decoding



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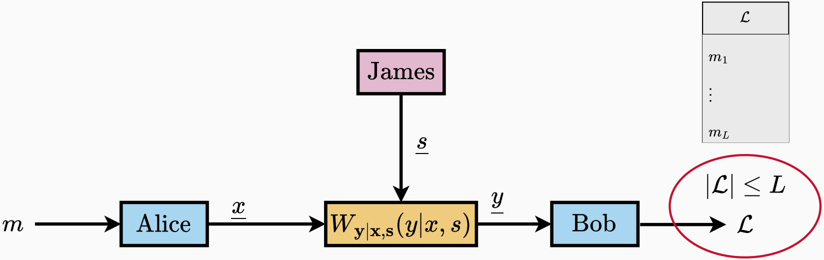

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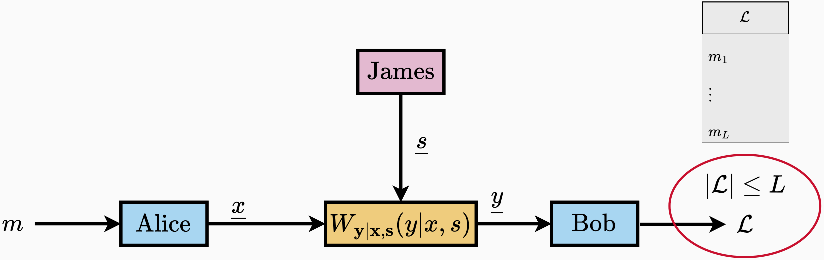

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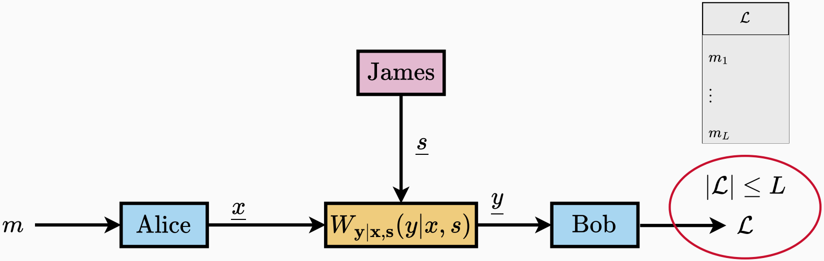

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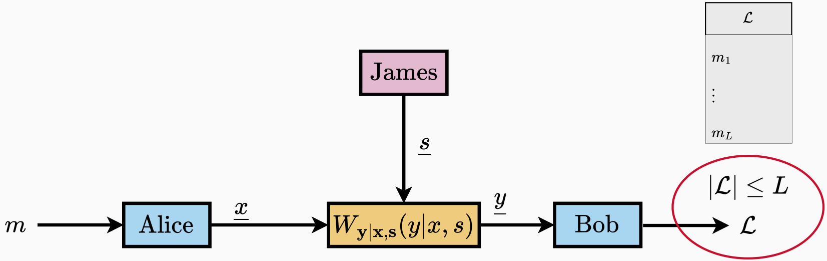

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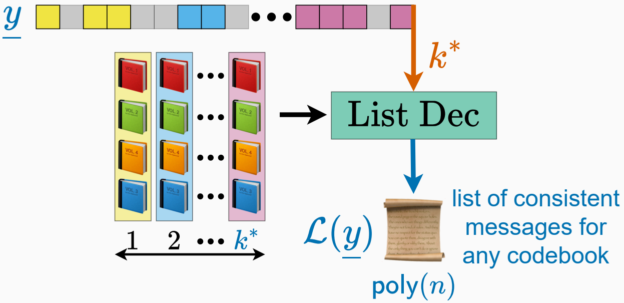
In some cases the list decoding capacity allows **strictly larger** rates:

$$C_{\mathrm{list}}(L) > C_{\mathrm{obl}}.$$

List decoding appears in many ways


$\underline{y}$
k^*
List Dec
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2
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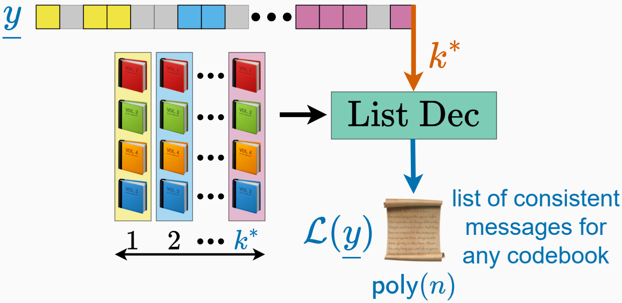

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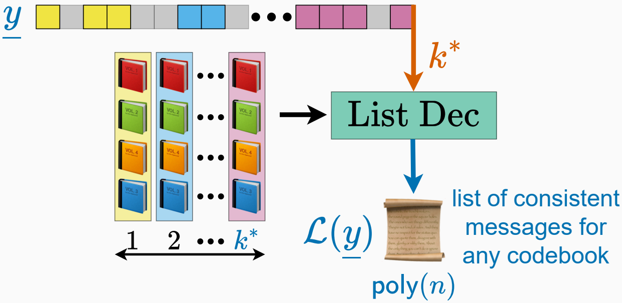
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List decoding to a “small list” as a first stage often leads to optimal decoders:

- With $O(\log n)$ bits of common randomness we get rates as good as infinite common randomness.
- James can list decode to jam more effectively.

Ingredient 3: Completely Positive (CP) Couplings and the Plotkin Bound

A more technical ingredient which is particularly useful is the notion of **completely positive couplings**.

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- A **self-coupling** is a joint distribution $P_{\mathbf{x},\mathbf{x}'}$ where each marginal is $P_{\mathbf{x}}$.

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- A **self-coupling** is a joint distribution $P_{\mathbf{x},\mathbf{x}'}$ where each marginal is $P_{\mathbf{x}}$.
- A self-coupling is **completely positive** if it is a mixture of independent self-couplings:

$$P_{\mathbf{x},\mathbf{x}'}(x,x') = \sum_{i=1}^{|\mathcal{U}|} P_{\mathbf{u}}(i) P_{\mathbf{x}_i}(x) P_{\mathbf{x}_i}(x').$$

Generalizing the Plotkin bound

Question: can we have a codebook where all codewords have pairwise types that are ρ-far from a CP self-coupling?

$$\|T_{\underline{x},\underline{x}'} - P^{(\mathrm{CP})}_{\mathbf{x},\mathbf{x}'}\|_\infty > \rho \qquad \forall \underline{x}, \underline{x}', P^{(\mathrm{CP})}_{\mathbf{x},\mathbf{x}'} \tag{1}$$

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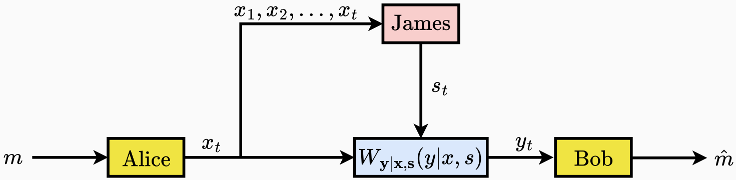
- It turns out that any codes with this property cannot be too large (for large n)!
- Compare this to the Plotkin bound: an upper bound on the size of binary codes with a given distance.
- If our rate is too high, then there will a constant fraction of codeword pairs whose type is close to CP.

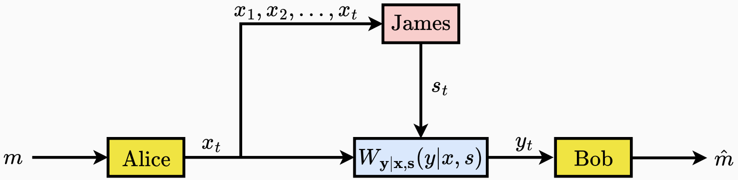
Causal adversarial models

Causal adversaries: James can see the current input



When can James “symmetrize” the channel and what does that mean? Think of James’s constraint as a “power limitation”:

Causal adversaries: James can see the current input



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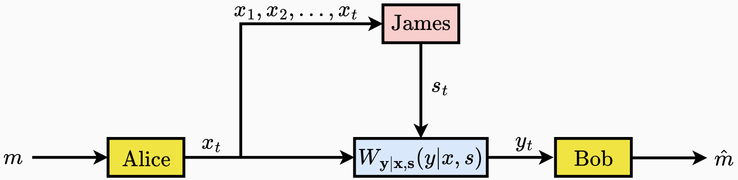

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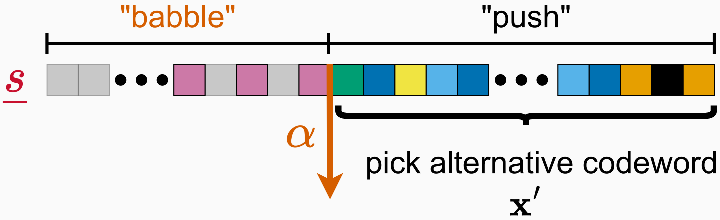
- Spend more power at the beginning in the hope of leading Bob astray? But then the suffix might resolve Bob's uncertainty.

Babble and push: an attack for James



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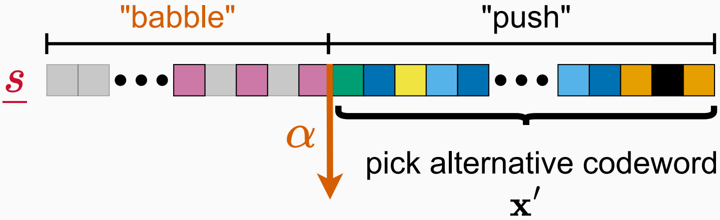

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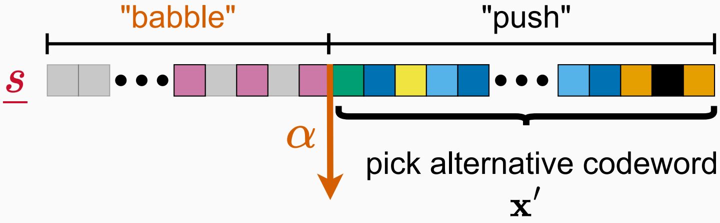

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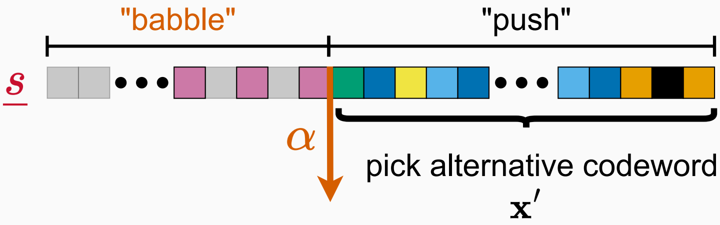

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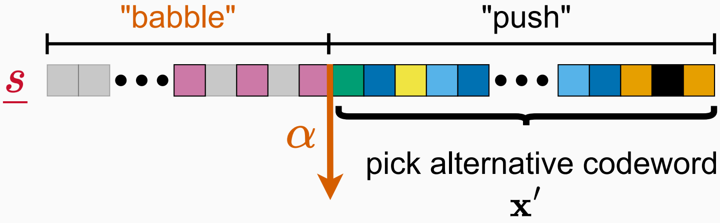

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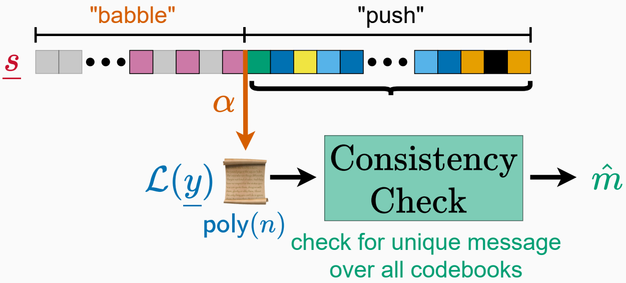
Use the generalized Plotkin bound (plus more) to show this will work.

Achievability



We can match the converse by using the same structure.

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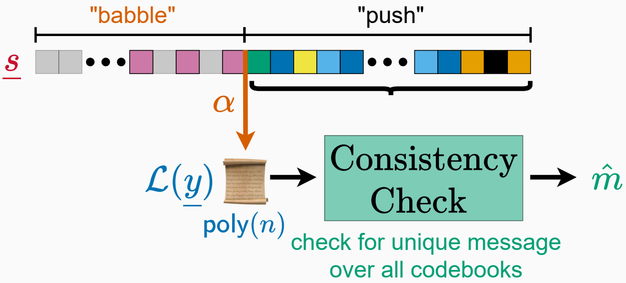

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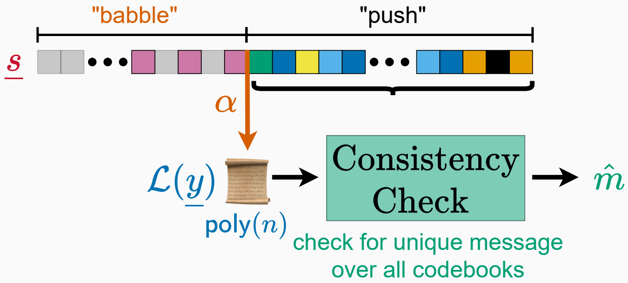

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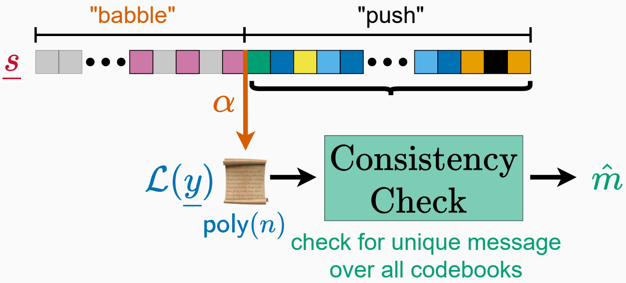

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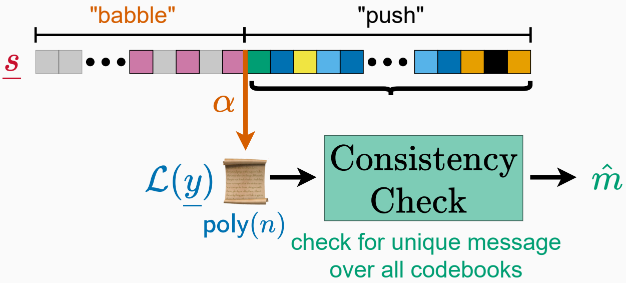

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Basically have to define what “feasible” means in this setting (quite involved).

A multi-letter block characterization

Pros and cons:

A multi-letter block characterization

$$C := \limsup_{K \to \infty} \max_{\substack{P_{\mathbf{x}|\mathbf{u}} \in \Delta(\mathcal{X}|[1:K]) \\ \left[\mathrm{Unif}([K]) P_{\mathbf{x}|\mathbf{u}}\right]_{\mathbf{x}} \in \Lambda_{\mathbf{x}}}} \min \Bigg\{ \min_{V_{\mathbf{s}|\mathbf{x},\mathbf{u}} \in \mathcal{F}(P_{\mathbf{x}|\mathbf{u}})} I(P_{\mathbf{x}|\mathbf{u}}, V_{\mathbf{s}|\mathbf{x},\mathbf{u}}),$$

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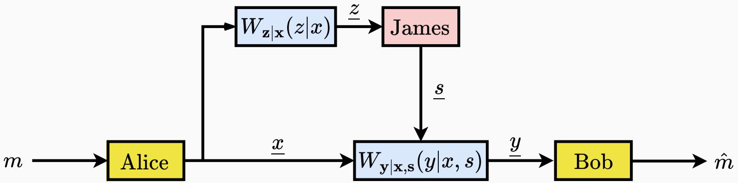
- ✗ Relies on some additional assumptions.

Myopic adversarial models

Myopic adversaries: James sees the whole codeword in noise



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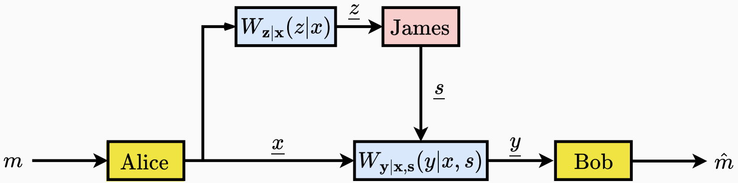

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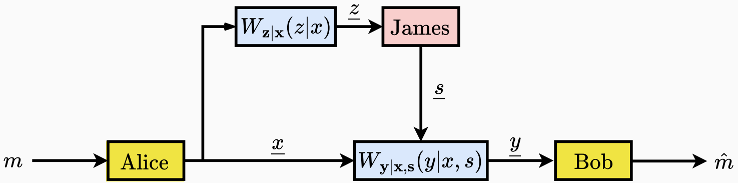

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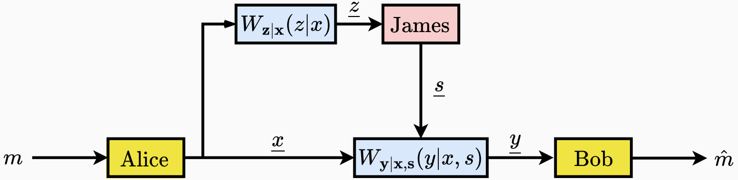

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- By changing $W_{\mathbf{z}|\mathbf{x}}$ we can get the oblivious and omniscient settings.

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Sufficient myopia and achievability

James can create an “effective DMC”

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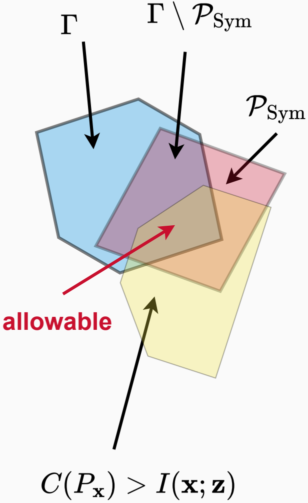
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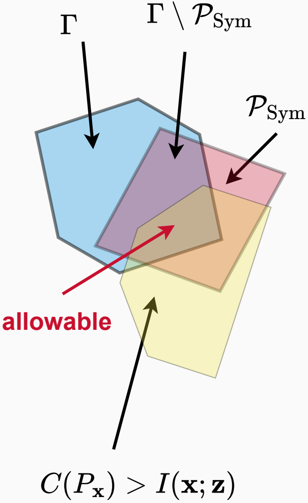
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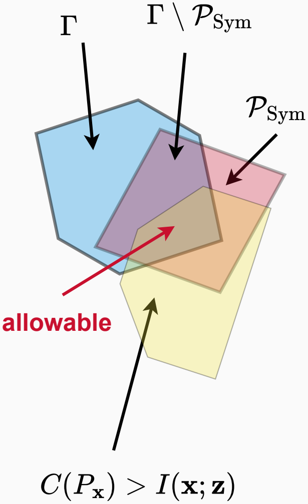
If $I(\mathbf{z};\mathbf{x}) < C(P_{\mathbf{x}})$ we say James is **sufficiently myopic**. In that case we can achieve any rate

$$R < \max_{P_{\mathbf{x}} \in \Gamma \setminus \mathcal{P}_{\mathrm{Sym}}} C(P_{\mathbf{x}}).$$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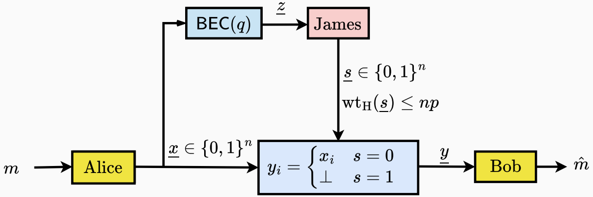
Myopic adversaries in the erasure setting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In the erasure setting the eavesdropping channel is a BEC(q) and James can erase at most pn bits. If $p < q$, James is **sufficiently myopic**.

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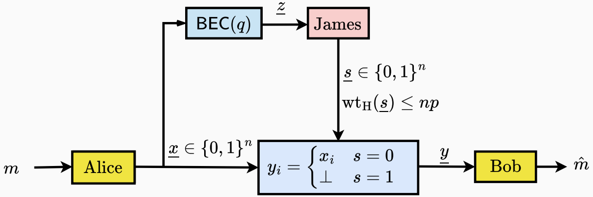
If $p < q$,

$$C_{\text{obl}} = 1 - p.$$



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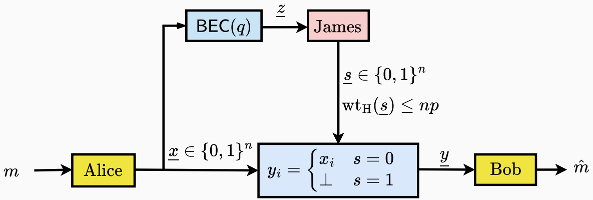

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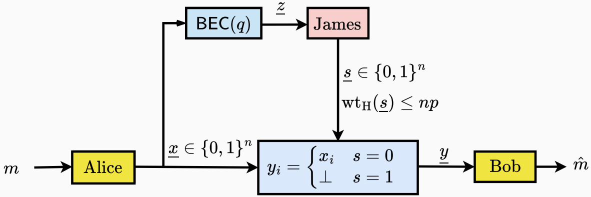
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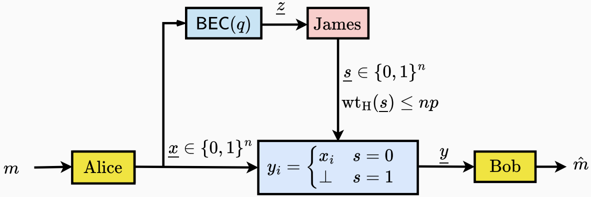
1. If $q > 2p - 1$,

$$C \in \left(0, (1-q)\bar{\alpha}\left(\frac{p-q}{1-q}\right)\right],$$

where $\bar{\alpha}$ is the LP bound for normalized distance.

Myopic adversaries in the erasure setting



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Bob
$\hat{m}$

In the erasure setting the eavesdropping channel is a BEC(q) and James can erase at most pn bits. If $p < q$, James is **sufficiently myopic**.

If $p < q$,

$$C_{\text{obl}} = 1 - p.$$

If $p > q$ we have two cases:

1. If $q > 2p - 1$,

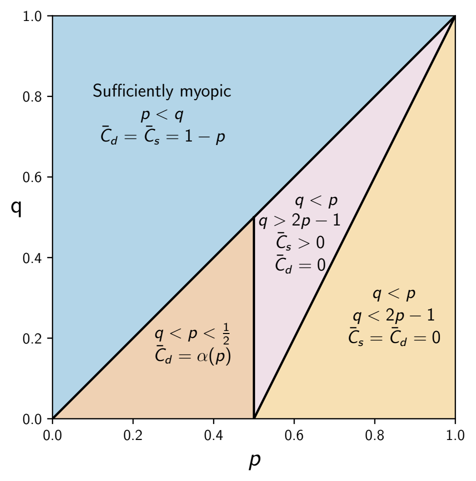
$$C \in \left(0, (1-q)\bar{\alpha}\left(\frac{p-q}{1-q}\right)\right],$$

   where $\bar{\alpha}$ is the LP bound for normalized distance.

2. If $q < 2p - 1$,

$$C = 0.$$

Myopic adversaries in the erasure setting



Computationally efficient codes for causal adversaries

Main questions we address

Can we design **efficient codes** **for causal and myopic models?**

By **efficient** we mean that they take **polynomial time** to encode, decode, and store.

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  $\longrightarrow$ use a **library of linear codebooks**.
- **common randomness** is unrealistic.
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- **minimum distance coding** is not efficient in general.
  $\longrightarrow$ use **list decoding** to permit **efficient decoding**.

“Efficient” coding schemes

To get **polynomial complexity**, use

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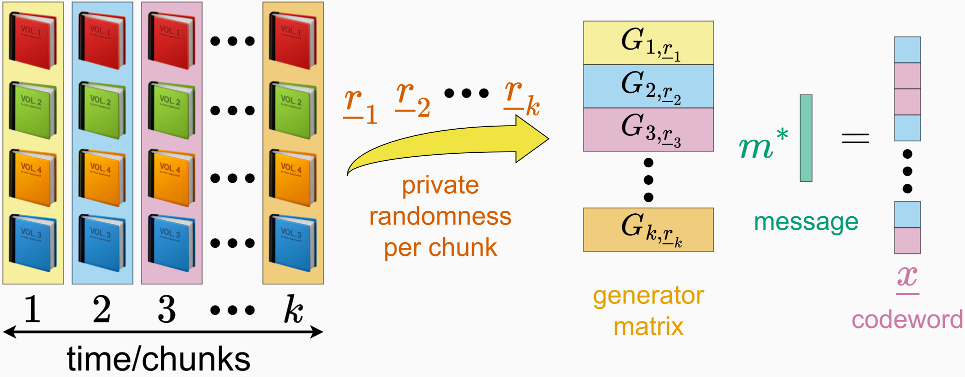
There are different types of complexity we would like to control:

- **Design**: how many bits do we need to generate the code?
- **Storage**: how many bits do we need to store the code?
- **Encoding**: how many operations are needed to encode a message?
- **Decoding**: how many operations are needed to decode the message?

Main results

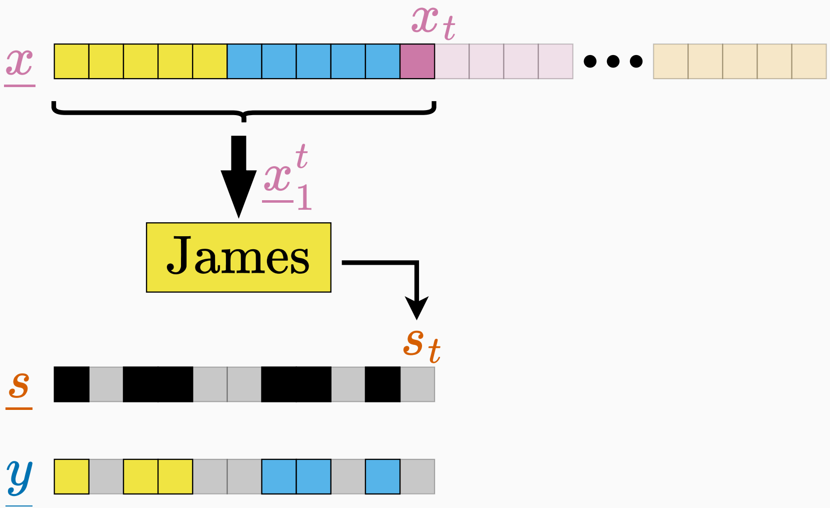
Model **rate**	**Randomness**	**Enc/Storage**	**Decoding**	$\mathbf{P}_{\text{error}}$
Myopic $p < q$ $\mathbf{1 - p - \epsilon}$	$\lambda_{SM} \log(n)$	$O(n^{2+\lambda_{SM}})$	$O(n^{3+\lambda_{SM}})$	$O(n^{-\lambda_{SM}})$
Myopic $q < p$ **small rate**	$O(n \log\log n)$	$O(n^2 \log\log n)$	$O(n^3 \log\log n)$	$O(n^{-4/5})$
Causal $\mathbf{1 - 2p - \epsilon}$	$O\left(\frac{\gamma \log n}{\epsilon}\right)$	$O(n^3 \log\log n)$	$O(n^{32/\epsilon})$	$O(n^{-(\gamma-1)})$

Encode splits block into a constant $k = \lceil \frac{n}{\epsilon} \rceil$ chunks

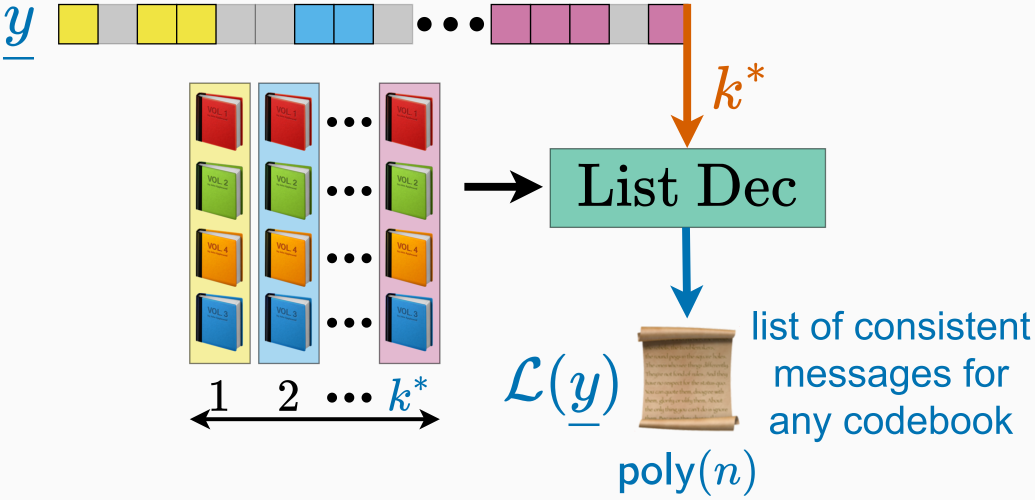


Generate a **library of linear codebooks** independently for each chunk.

James can erase with causal information only



Bob decodes to a polynomial list



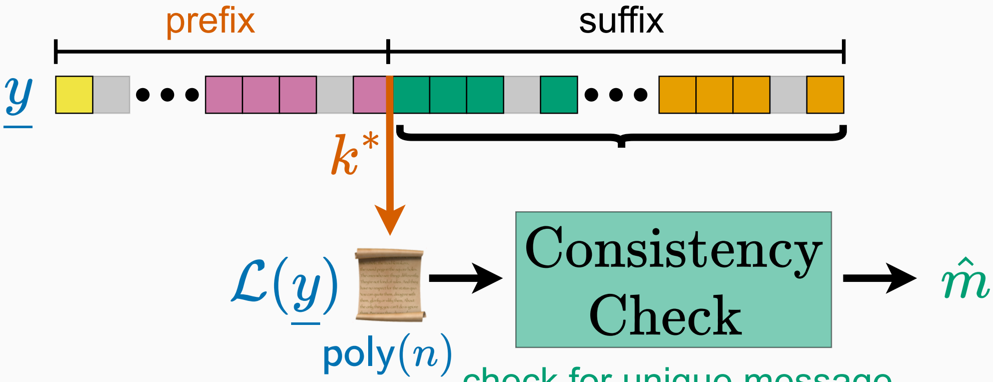
List Dec

$\mathcal{L}(\underline{y})$

list of consistent messages for any codebook

poly(n)

Bob uses suffix to disambiguate the list



prefix

suffix

$\underline{y}$

k^*

$\mathcal{L}(\underline{y})$

poly(n)

Consistency Check

$\hat{m}$

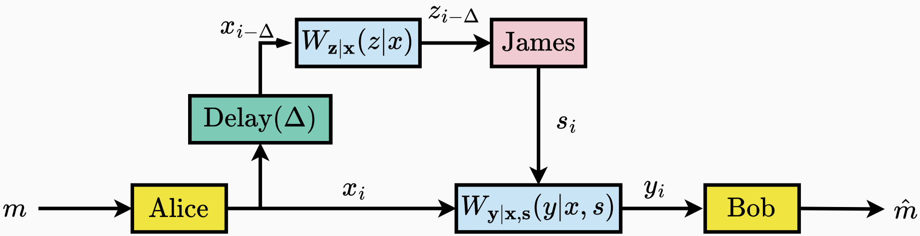
check for unique message over all codebooks

Why does this work?

1. Bob can **track James's erasure budget**.
2. List decoding creates **a smaller set of messages** to check for consistency.
3. James has a choice to **make the list larger** (erase more earlier, less later) or **conserve his budget** (erase less earlier, more later).
4. **Poor James, he can't win.**

Looking forward

Other intermediate models



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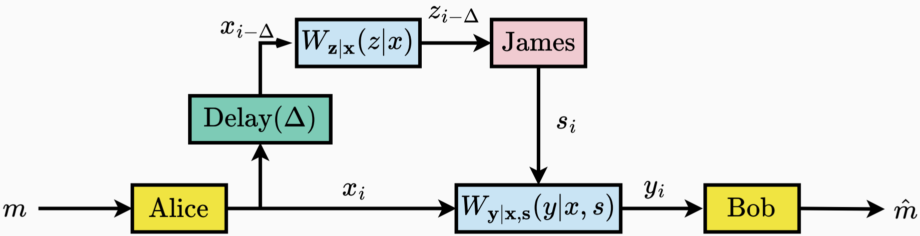

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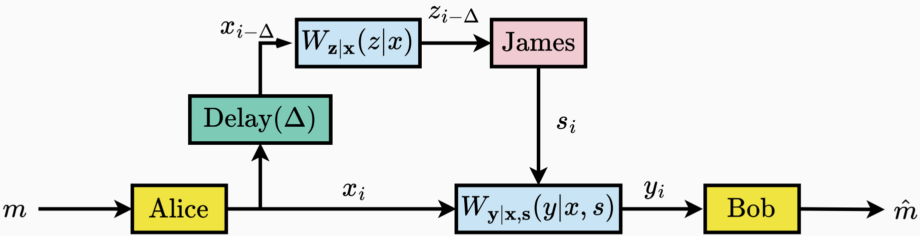

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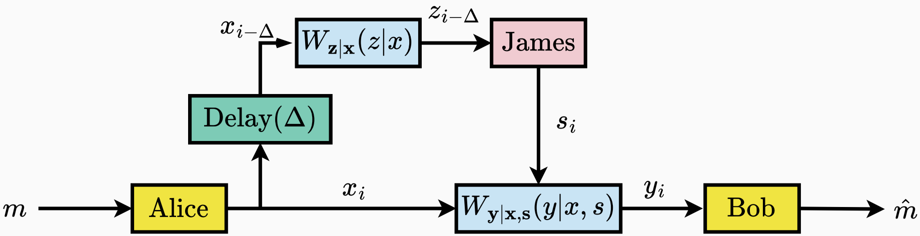

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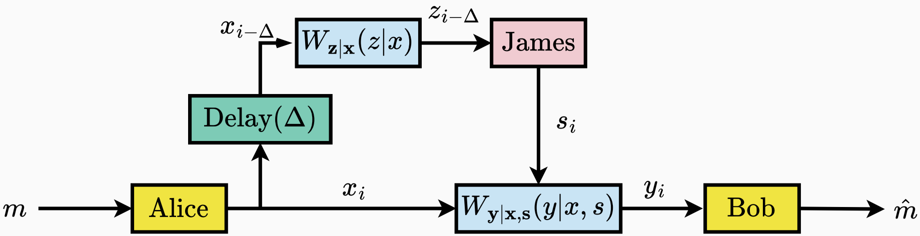

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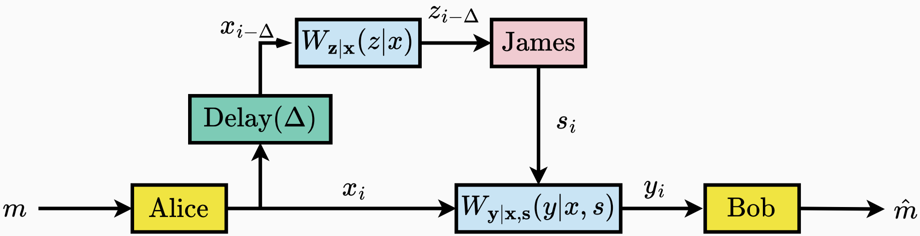

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Each model will reveal something about what the **worst-case channel** looks like.

And for the theory folks...

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Understanding AVCs has lots of connections (perhaps less well described here) to many interesting areas:

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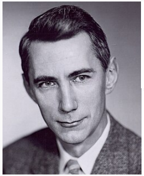
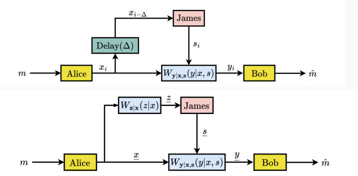
- adversarial machine learning
- extremal graph theory
- other fun combinatorial problems

A final recap and takeaways





AVCs can capture models between average and worst-case channels.

- **Causal:** capacity depends on **what James knows about the current input.**
- **Myopic:** capacity depends on **whether James can (partially) "decode."**

Some insights:

- **Stochastic encoding** and **list decoding** can help!
- Adversarial attacks are more powerful **at the end of decoding**.

Thank you!